PUSE Study

Poly-Universe in School Education

Erasmus+ PUSE Project

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Project Coordinator:

Polyuniverse Ltd., Hungary

www.poly-universe.com

PROLOG

Present-day children have a new relationship with the world: they are the citizens of the ’global village’. They live in an unlimited network of connections, synergies and crossings. The whole system of international education, teacher training and teachers have to face great challenges. They want to find new answers to new questions.

The struggle between the teacher’s expectations and the pupil’s desire to meet them is step by step pacified and may culminate in a fruitful cooperation. Every teacher in the world is motivated to achieve this and is seeking methods and means to help them. The subject of our Erasmus+ application, the pedagogical methodology developed from the Poly-Universe skills development tool shows a solution in this process. We have chosen four partners from various parts and educational systems of Europe: a Finnish, a Hungarian, a Slovak and a Spanish one. Over the past five years, considering mainly the workshops held in schools, an international need has emerged to use the methodology developed through the application of Poly-Universe in improving a new type of skills, abilities, knowledge of the future generation, to have an innovatory kind of education of mathematics. Our project presents and makes available for schools in the EU a pedagogical practice based on geometry as universal visual language, is experience-oriented, involving each sense organ, improves social abilities and explores the synergies of science and arts.

Non-Euclidean geometry and the geometric pedagogical method built on this have not spread in the mainstream European teaching practice. Teaching Mathematics based only on calculation has become the norm and has not been questioned by generations of teachers. The European way of teaching geometry has said for centuries that Euclidean geometry is to be limited to a row of theorems to be validated using lengths and angles turned into numbers, with algebraical instruments. This approach destroys the view in children’s minds and makes it almost impossible to understand and grasp non-Euclidean geometry indispensable in modern sciences. Teachers of Mathematics working in higher education can tell how much damage this unilateral approach has caused. However, this situation can be changed.

Western geometry-teaching forces the human brain in the wrong direction. Algebra is a science of rational thinking using the left hemisphere of the brain, building on the sequence of logical conclusions. Geometry should be a science using the right hemisphere, based on stereoscopic vision and a holistic, intuitive way of thinking. The algebraized method of teaching geometry, which deforms the shapes into metrics describing them, gives priority to rationally thinking children; while pupils with a right hemisphere dominance, thinking intuitively in space and sets, cannot succeed. They find geometry lessons burdensome and their inherent abilities and artistic talent will wither during the school years.

It is not by chance that the left hemisphere of the brain is often referred to as the 'talking' hemisphere, while the right hemisphere is called the 'seeing' hemisphere. Neither is the fact that the idea of this educational tool, which is based on view and on the geometric features of shapes was born and developed in the brain of a most truly artist János Szász SAXON.

Poly-Universe is the source of shared experiences and recognitions, a means of joyful creation. Besides promoting talent and teaching mathematics and art, it can vent the frustrations of underprivileged or less talented pupils, making them perform better; thus, changing the community's fair or unfair sociometric classification. The tool is especially suitable for developing abilities of children with various disabilities (autistic, deaf and hard of hearing, blind and visually impaired ones). Poly-Universe not only proved to be a skills development tool or an aesthetic game, nor simply a range of philosophical, mathematical and combinatorial activities, but all these together. In the PUSE project, together with teachers and pupils of European partner schools, we developed a new visual mathematic educational system, the methodology of synergies. This knowledge is passed on through our website available for EU schools in the future.

So far, expert opinion has made it clear, and the achievements during the implementation of our project in the future will also show that PUSE methodology provides the same comprehensive structure change in thinking in educational systems of countries both in Europe and in any part of the world.

Therefore, we have sought forward-thinking educational experts for collaboration who, having understood the message of our time, interpret mathematical and geometrical knowledge and works of art as bridges between the abstract shapes of knowledge and perceptible forms of creativity; who keep looking for links, passages, playful opportunities for education using an inter-art and interdisciplinary approach.

The PUSE Study is the first main intellectual output of our project. This study will deliver a complex methodology about the use of Poly-Universe as a skill development teaching tool in school education. This study is the foundation of developing other intellectual outputs: A Teacher's book (analog), an Exercise Book (analog) and the PUSE Online System. The PUSE Study will consist of six parts, six separate studies from our six Partners implementing Poly-Universe's geometric skill development tool in various subjects (geometry, mathematics, combinatorics, development pedagogy, talent management) and for different school ages (6-10, 10-14, 14-18). This pre-methodology involves the experiances of all pedagogues, psychologists, sociologists, art curator, artist/inventor from the Partner Institutions.

Zsuzsa Dárdai
Project Coordinator

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The project was supported by the European Commission. Published in the Communication does not necessarily reflect the views of the European Commission.

Poly-Universe Ltd

The coordinator and owner of the project is Poly-Universe Ltd. led by the inventor of the game: János Szász SAXON. The company owns the basic products and related ideas. Since these inputs are indispensable for using the game in education, it is necessary that the company implements the program at coordination level and becomes responsible for project management.

Poly-Universe Ltd. was established in 2010 with the purpose of creating the Poly-Universe geometric skills development game family. The company also received international trademark protection for Poly-Universe. The game is manufactured by the Hungarian Gyurik Plastic Company, but Poly-Universe Ltd. is responsible for the total quality management.

Poly-Universe games have been tested for five years by the inventor and by the pedagogic coordinator Zsuzsa Dárdai. They have organized performances and workshops in nearly 100 schools where they gained positive experiences without exception. The demand encountered during these workshops in Hungary and abroad gave them the impulse to outline the methods necessary to introduce the invention to public education within the framework of the tender.

As for the selection of partners, the guiding principles were the openness and enthusiasm of the foreign institutions acquainted during the workshops in the previous years. This provided the first circle of partners to call on. Further, it has become their aim to expand the existing system of relations into an educational network. The circle of partners has therefore been extended with institutions recommended by professional institutions and experts involved in the project. In finding partners, they have tried to select the widest possible range of public education, covering primary (lower and upper section) and secondary schools and students. In order to implement the project, students are provided by the partner schools selected for the project.

The participating students and teachers are going to take part in testing and evaluating auxiliary training materials under development, both paper-based and in digital format.

Besides of choosing school partners, it has been their consideration to involve a research institute, technical organizations and experts. In case of the selected schools it has been vital that they should be educational institutions concerned with and committed to Mathematics.

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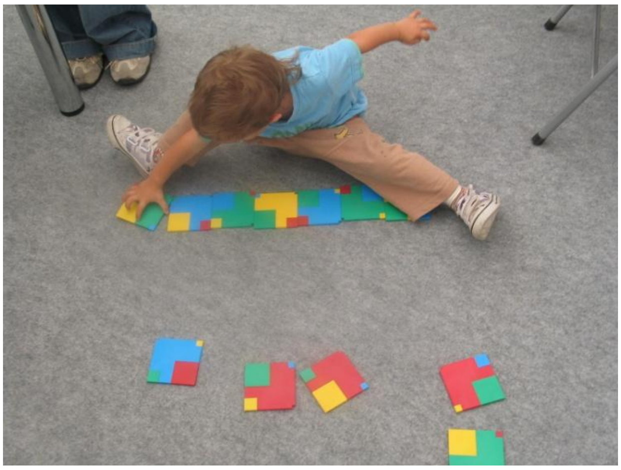
CONTENTS:

I. POLY-UNIVERSE as an Endless Educational Option

The game family, in using and developing the basic geometric shapes of János Szász SAXON's poly-dimensional plain painting, communicates a new artistic perspective to both nursery and primary school children and adults. Having a direct, by-touch connection with the geometric shapes, the sense of vision and of touch are developing. Also, by recognitising correlations and finding linking points, the ability of thinking improves and abstraction skills evolve. The learning process:

- Sight
- Touch
- Detection
- Perception
- Memory
- Attention
- Concentration
- The discovery of part-whole
- Imagination
- Creativeness
- Problem-solving

These colourful and easy-to-handle geometric shapes generate infinite logical complexity and complex mathematical and morphological puzzles. However, its strength is its simplicity as it provides an equal opportunity for children of different ages, at different levels of mental and emotional maturity to develop their personality.



Progressive use of colour and shape groups and the high degree of manual activity and reflective thinking create a constant challenge for children and maintain their desire to explore, at the same time allowing an undisturbed and continuous feeling of success. Direct physical activity, the emotions conveyed by colours, and the possibility of trying out plenty of variations without being checked make the children feel free and relaxed. Such experiences help children to find more creative and imaginative solutions when dealing with problems from different areas of life or acquiring new knowledge.

Thinking operations development:

- Analysis
- Synthesis
- Abstraction
- Comparison
- Perception of correlations
- Generalization
- Clarifying
- Analogy
- Order

All in all, “POLY-UNIVERSE” should not be limited to school lessons, nor to after-school problem-solving classes, nor to any controlled activity. While being suitable for all these, it can act as a catalyst for the new pedagogical practice of learning by playing: teaching to see. The game family did not set up the rules for the game. During the workshop the children and adults will not solve given mathematical problems, but they will recognize the mathematical and aesthetic correlations hidden in the system individually.

These correlations could be summarized as follows:

- Discovering geometric shapes
- Searching for proportions
- Examining symmetry
- Finding the linkage points
- Setting the directions
- Making colours collide
- Mixing forms
- Expanding the limits of composition
- Possibilities of combination
- Feeling the infinite

So, this game family does not only aim at problem-solving or recognizing colours or shapes, or solving logical puzzles. It also offers the possibility of playing a game freely, so children or adults can learn an activity indirectly, through a game. When dealing with the different scale basic geometric shapes and primary colours, they gain experience, discover and see correlations, points of linkage and shape connections and the sharp borderlines between colours, not knowing that they are learning.

They can explore “POLY-UNIVERSE”, the realms of mathematics, art and philosophy, wandering engrossed in them, without being aware of where exactly they are. This novel game family does not only develop skills or offer a visual or aesthetic experience; it also expands scientific knowledge as it is based on an extraordinary mathematical set of proportions: scale-shifting symmetry.

Children and adults of different age groups, culture and social background can take part in the workshop. Everybody can enjoy the game during the workshops, whether disabled or not, whether children or adult.

The benefits of the game:

- Developing logical, pedagogical, mathematical and creative skills,
- Personal growth
- Impact
- Indirect learning method
- A new vision of art forms through the geometric poly-dimensional plain painting
- Arousing the desire for exploration
- Continuous sense of achievement
- Team work opportunities, developing cooperation and the need for team spirit
- Infinite number of combinations
- Cognitive development helping children's learning processes

II. RECOMMENDATION

We have tested the receptivity of the Poly-Universe skill-developing tool from nursery school to university and we have found that the game includes the adaptation to age-related idiosyncrasies. By recognizing the colours and the sizes of the forms, three-year-olds may play intensely with the tool for a long time and in each case the "aha! experience" appears on their face as the sign of success, which is important in personality development.

For primary school, high school and university students or even professors, the hidden possibilities of the game remain: basic forms, basic colours, coupling points, logical, mathematical-combinatorial relations, aesthetics, natural scientific thinking, philosophical thinking, etc. The method of finding solutions, however, is palpably follows the revealing characteristics of the age-related idiosyncrasies, the level of knowledge and the focus of recognition.

Due to its disadvantage-compensating and skill-developing values, the suitability of the Poly-Universe game to failure-oriented children was also proven throughout the years. Often, a generally underestimated "bad student" of the class could experience a so-far unfamiliar, great success by winning in a test-free task solving activity. Based on the feedback by the teachers, such students later "animated", they just "flew forth" on the way of acquiring knowledge.

In the use of the Poly-Universe game there is nothing like "meeting requirements". No matter who is poor and who is rich; who is smart and who is dull; who is disabled and who is healthy. Everybody faces the challenges of the game with equal chances. The game is an eternal basis - its reception is always changing.

As a skill-developing tool, Poly-Universe is the supporting feature of the reformation-longing pedagogical methodology, which continuously cooperates with teachers who are committed to adventure learning, artists, mathematicians and children.

For school methodology, we have chosen our tasks in a way that the personality-forming process of playing with Poly-universe and its treasury of possibilities that are open to all age-groups becomes sensible. For instance, lower graders play with only 2-3-4 elements and recognize colour and form relations of certain form-packages. Upper graders work with more elements, examine their closed system of relations while high school students are already able to handle the entire set, create and solve combinatorial tasks and formulas.

III. GLOSSARY of PUSE methodology

1. **BASE FORM:** circle, triangle, square

2. **BASIC ELEMENT:** a basic object designed from the base forms with shapes of various size and colour.

3. **COLOURS:** red, yellow, blue, green

4. **BASE COLOUR:** the colour of the central part of the given basic element

5. **PACKAGE:** game package of 24 basic elements

6. **GAME FAMILY SET:** a complete set of 3x24 pieces of various basic elements (circle, triangle, square)

7. **SIZE:** (abbreviated by B; L; M; S; H)
   - B=basic element
   - L=large
   - M=medium
   - S=small
   - H=hole (in case of squares)

8. **CONNECTION POINTS:** matching the sides or vertices of the smaller shapes on the vertices of the basic elements, based on some regularity (colour or size). In case of circles we take the diameters of the semicircles and their "midpoints" and "vertices" (endpoints of the diameters). These can be connected by:

   - **TOTAL SIDE CONNECTION**: the basic elements are connected with the whole length of their sides in case of triangles and squares. With circles, the diameters of the semicircles of the same size are connected.

   - **SAME COLOUR:** only the shapes of the same colour are connected (there may be some of the same size among them)

   - **SAME SIZE:** only the shapes of the same size can be connected (there may be some of the same colour among them)

   - **SAME COLOUR and SIZE:** only shapes with the same colour AND size can be connected.

- **VERTICES:** the vertices of the basic elements are connected

- **SLIDING:** the basic elements are slid along the sides (thus not the entire sides of the basic elements are connected) and the smaller elements are connected to each other based on some regularity (at least one vertex on each of the matching sides). In case of circles we can connect semi-circles of various sizes, then the "vertices" and midpoints of the semicircles can be the connection points in any combination (vertex-vertex, midpoint-vertex, midpoint-midpoint).

9. **CLOSED SHAPE:** shapes created with total side connection, listed by basic element types:

   a. Base form triangle
      - bigger triangle of 4, 9, 16, 25-1=24 basic elements (in the latter case there is a hole in the middle)
      - rhombus, trapezoid, parallelogram
      - hexagons of 1x6, 2x6, 3x6, 4x6 basic elements, or the biggest hexagon of 24 basic elements

   b. Base from square
      - bigger square of 4, 9, 16, 5x5-1=24 basic elements, in the latter case there is a hole in the middle. With a double package a bigger square of 7x7-1=49-1=48 basic elements (with a hole in the middle) is also possible.
      - rectangle of 6x4=24 or 6x8=48 (double package) basic elements

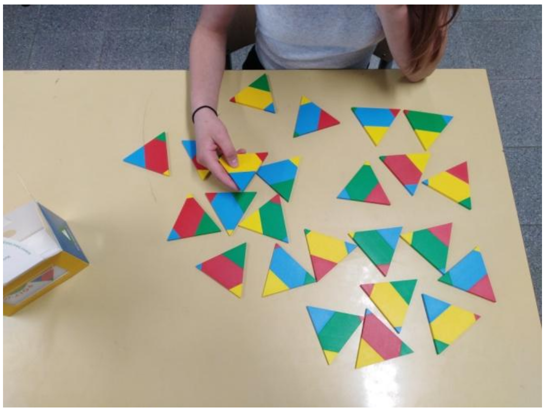
   c. Base form circle
      - ring from 6 basic elements constructed by total side connection.
      - connected shapes of 2, 3, 4, 5, 6, 7 rings. With a double package a shape of up to 13 rings can be completed.

10. **OPEN SHAPE:** Any other shape: with branches; an open chain (in case of circles); a shape constructed by sliding and vertex connection; or mixing the elements of various base forms.

IV. GENERAL Geometric Tasks: form and colour mergers

IMPACT: Sight and touch, Form and colour senses, Area calculation (based on estimation), Abstractive vision, Art sensitivity, Complex logical thinking, Combination skills, Attention and concentration, Invention, Memory, Courage, Openness and team spirit...

TASK 1/ Put all elements of the kit on the table and examine them (triangle, circle or square)



TASK 2/ Estimation: which of the different smaller forms (large-mid-small) do you have the most on the table? Count them.

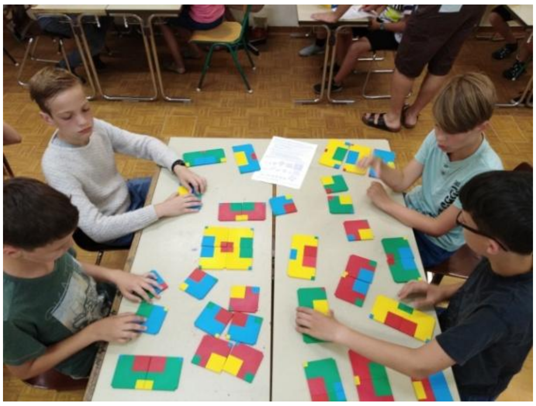
Solution: same amount - 24 large, 24 mid-size and 24 small

TASK 3/ Select the smaller sized forms according to colours. How many identically coloured smaller forms do you find? Count them.
Solution: identical amount, 6 pieces of small, mid-size and large per colour

TASK 4/ Compare the areas of forms with identical colour and size (e.g. small red – small blue, medium-size green – medium-size yellow etc.)
Solution: these are also identical.

TASK 5/ Consider the entire toolkit and estimate, which of the colours (red-green-yellow-blue) has the largest area? *Solution: the areas are identical*

TASK 6/ Pick an arbitrary number of basic forms from the kit and join them so that only the identically coloured segments of the sides connect (triangle or square). Do you think that basic forms might connect along their full-length-sides as well? Why?



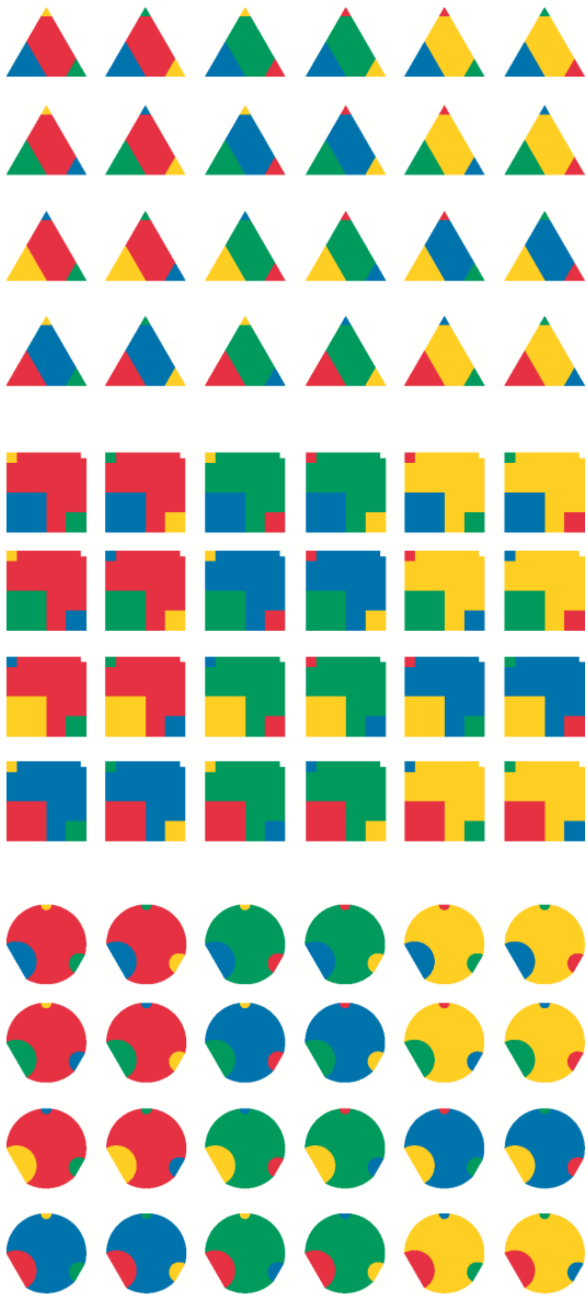
TASK 7/Relying on the previous rule, join all the 24 basic forms. How much time did this take? Estimate how many solutions there can be.

Solution: about 10 min, practically countless possibilities

V. GENERAL Combinatoric Tasks

In this chapter we enumerate the main attributes and rules of the POLY-UNIVERSE from the mathematical point of view. The selected inspection method is combinatorics since this branch of discrete mathematics is suitable for discovering the number of possibilities inherent in the product family in the most comprehensible and effective way.

All considerations described here are plausible, but the number of the joint possibilities explained below is many times surprising. The aim of this chapter is not to examine the POLY-UNIVERSE closely and from all possibles directions. This goal is hindered by the limits of its extension and by the high number of approaches. At the same time, we undertake the challenge to feature the fundamental correlations of the game family we find important.



Examination of the set, permutations

TASK 1/ The inventor developed the packages containing the toy elements by using the above formal and colour building elements so that all of the elements are different. How many items are in the set? Why so many? How can it be calculated?

Solution: The number of the elements in each package presents itself through the simple permutation without repetition of the four colours in all of the three cases, i.e. each package contains $4 \bullet 3 \bullet 2 \bullet 1 = 4! = 24$ elements.

TASK 2/ Calculate that if we wouldn't cut out the peaks of the square, how many different elements could you get? How much could it have if we were to do the same thing with pentagons?

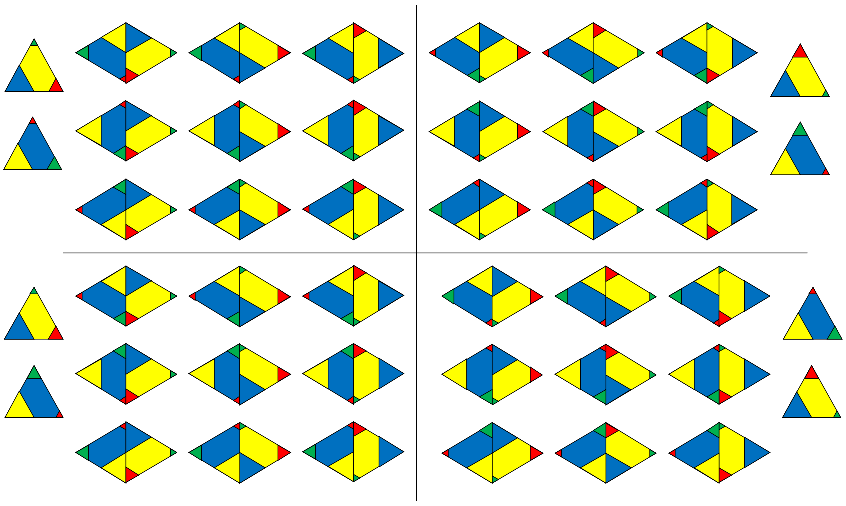
Solution: In this case, the number of the elements in each package presents itself through the simple permutation without repetition of the five colours in all of the three cases, i.e. each package contains $5 \bullet 4 \bullet 3 \bullet 2 \bullet 1 = 5! = 120$ elements. For example, in the case of a pentagon, we would break down all the basic elements into 6 parts, so we would have to assign 6 different colours. That is, $6 \bullet 120 = 720$ different basic elements would be created.

Entire side joints of triangles

TASK 3/ How many possibilities are to connect two triangle basic elements on their whole side to get different shapes each time?

Solution 1: A triangle can be placed in six different ways. A second triangle can also be put down in 6 different ways, so $6 \bullet 6 = 36$ different possibilities turn up by the arrangement of two triangles with entire side joints.

Solution 2: We get the same result if we examine that two triangles turned up with the given faces can be placed in 9 different ways related to each other, since we can match the three sides of the first triangle with three different sides of the other triangle. Nevertheless, both triangles can be placed in two ways (basic and inverse position – see above) on the table. Of course, we get the above result in this way too: $9 \bullet 2 \bullet 2 = 36$.



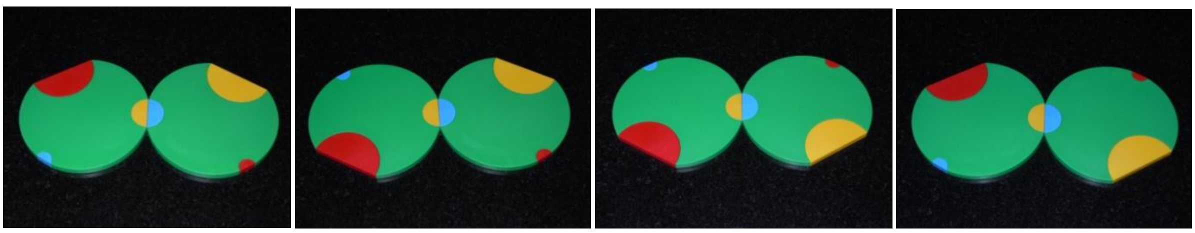
TASK 4/ In how many ways can we connect more than two (finally 24) triangle basic elements with connection of total sides, so that in each case we get a different layout, when we make a long line of elements. How does the solution change if any newly deposited triangles fits into any free side of a previously deposited shape?

Solution 1: Let us proceed in the following way: one triangle can be set down in six ways on the tabletop. Following that, for example, by putting down all of the 24 triangles we can get $6^{24} \approx 4{,}74 \bullet 10^{18}$ different configurations without changing the place of any triangle within the configuration.

*Solution 2: As long as we examine how the subsequent triangles relate to each other, the ollowing rule is true: $(n+1)! \bullet 3!^{n-1}$, where **n** is the number of the triangles. Adopting the formula for two triangles, we definitely get the 36 possibilities already calculated in two ways earlier. But in case of three triangles we get 864 different set-ups in that way. If we use all of the 24 triangles, $1.2 \bullet 10^{43}$ different possibilities would already turn up.*

Entire circle joints

TASK 5/ How many possibe ways are there to connect two circle basic elements with their total side, to get different shapes each time?



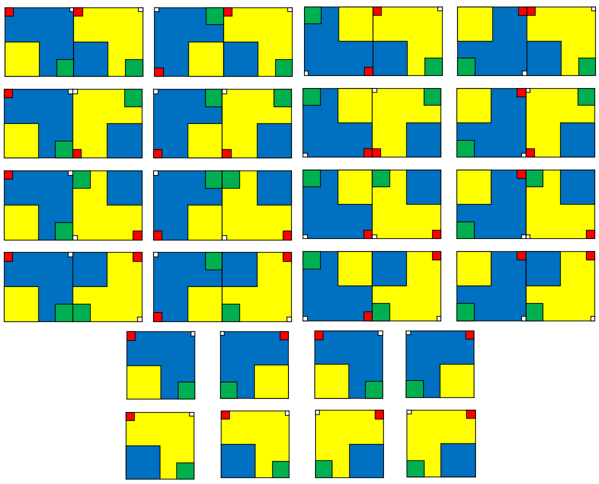
Soultion: In this way two circles can be set in four ways, and this is true for all of the three different sized semicircles. This results in $3 \bullet 4 = 12$ ways of placement of entire circle joints related to each other.

TASK 6/ In how many ways can we connect more than two (finally 24) circle basic elements to be connected along total sides, so that in each case we get a different shapes, when we make a long line of elements?

Solution: If we set a circle on the table, the second circle can be set next to it in six different ways with entire circle joints, since all of the three different sized semicircles are free, and the next circle can be set in two different ways on all of the three places. If we go on combining so that the next circle will be joined to the circle just put down, we only have two free semicircles, and the subsequent circle can be joined to them in two ways. That means that the set-up sequence can always be continued in four ways starting from the third circle. That also means that we get $6 \bullet 4^{22} \approx 10^{14}$ *possibilities by placing the 24 elements set in this way (as if they were in one row).*

Entire side joints of squares

TASK 7/ How many possibilities are there to connect two square basic elements with entire side joints to get a different shape each time? In how many ways can we connect more than two (finally 24) square basic elements to connect with their entire sides so that in each case we get a different shape when we make a long line of elements?



Solution: Two squares can be placed in 64 ways next to each other. This can be easily seen with the extension of the logic used with triangles, we consider the different number of sides, since a square can be put on the table in eight different ways.

1.	*2.*	*3.*	*4.*	*5.*	*…*	*24.*
8	*8·8*	*6·8·8=2·3·8·8*	*(2·4)·(2·3)·8·8*	*(2·5) ·(2·4) ·(2·3) ·8·8*		*(2·24) ·(2·23) ·…(2·3) ·8·8*

Then, in case of 24 squares we have $8^{24} \approx 4{,}72 \bullet 10^{21}$ *placing opportunities if we do not examine the location of each element in the configuration, plus if we set aside the closed arrangements.*

TASK 8/ Master: How many possibilities are there to connect the elements when examining the position of each element within the configuration? *Solution:* $8 \cdot 2^{23} \cdot 24! = 4{,}2 \cdot 10^{31}$

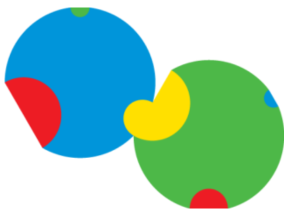
Slidings of the basic elements

TASK 9/ In how many ways can you connect two triangular elements to one another if you slide them? Then, the basic elements are slid along the sides (thus not the entire sides of the basic elements are connected) and the smaller elements are connected to each other based on some regularity (at least one vertex on each of the matching sides).
Solution: Nevertheless, the possibilities of the slidings are a bit more complicated than it would seem at first sight since the number of possibilities depend on the sides we join to. The triangles can be slid along the different sides as shown in the table above. ("R" means the short-, "K" means the middle- and "H" means the long sided internal triangle).
This is a symmetric matrix for the reason of the axial symmetry of the reverse elements.
One of the maximums is explained here as an illustration, for example the 14 sliding possibilities of the H-R – K-R sides:

One of the minimums explained as an illustration, for example the 7 sliding possibilities of the H-R – R-H sides:

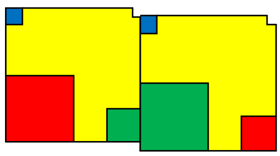
TASK 10/ Master: These slidings increase the above calculated already large number of the different configuration possibilities remarkably. Therefore, the question is how many ways you can put more than two (finally 24 triangle elements) in a chain with offset in order to get a different layout in each case?

TASK 11/ In how many ways is it possible to join two basic circle elements in a way that the vertices of a larger and a smaller semicircle fit to each other?



Solution: By this method it is possible to join two circles in 15 ways, and this number is multiplied by four because of the difference between the basic and inverse positions. So we have 60 different possibilities by this time, i.e. we can set up so many different configurations in case of two circles with partial joints.

TASK 12/ In how many ways is it possible to join two basic square elements by sliding so that at least one vertex per element fits to the other?



Solution: The slidings of the squares are analogous to the slidings of the triangles along the stable side linkage points. The difference is that the number of the sliding possibilities varies between 4 and 9. Inspired by the previous example we set two squares next to each other, and even if we calculate just with the minimal sliding possibilities by the side-pairs, we get 64 • 4 = 256 different configurations.

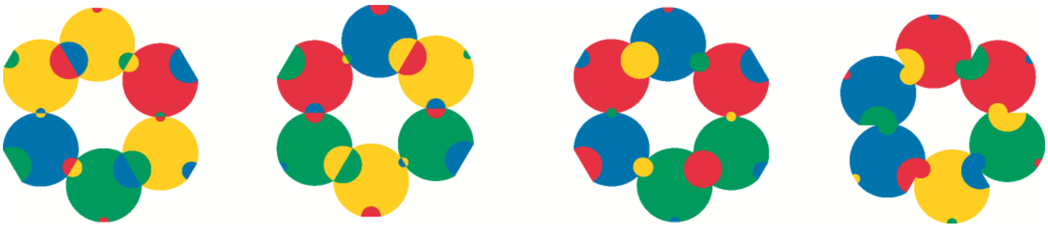
Closed Shapes

TASK 13/ How many possibile ways are there to select 6 triangle basic elements randomly from the 24 pieces set to get different constructions of closed shapes?

Solution: This means the six combination of the 24 elements which can be calculated as follows: C^6_{24} = 134596. After the selection we have to the number of different arrangements of the next triangle according to the already placed triangles in the course of our attempt to get a closed configuration. (The numbers under the factors below mean the ordinal number of the placed triangle.) Thus, we get a different configuration of an order of magnitude of 100 billion.

$$C_{24}^{6}\cdot\underbrace{(1\cdot6)}_{1.}\cdot\underbrace{(3\cdot6)}_{2.}\cdot\underbrace{(4\cdot6)}_{3.}\cdot\underbrace{(2\cdot6)}_{4.}\cdot\underbrace{(2\cdot6)}_{5.}\cdot\underbrace{(1\cdot6)}_{6.}=C_{24}^{6}\cdot48\cdot6^{6}\approx3\cdot10^{11}$$

TASK 14/ In how many possibile ways can we select 6 circle basic elements randomly from the 24 pieces set to get different rings of closed shapes?



Solution: The formula above alters in case of circles in such a way that instead of the factor 6^6 that represents the arrangement possibilities of each element we have to consider 2^6 or10^6 possibilities depending whether we allow entire circle joints exclusively or partial circle joints as well. In the latter case the above product increases the order of magnitude by one, whereby we step into the range of 10^{12} (trillion).

TASK 15/ In how many possibile ways can we select 4 square basic elements randomly from the 24 pieces set to get different bigger squares of closed shapes?

Solution: In case of four squares we consider the square arrangement next to each other as closed configuration. If we accept only entire side joints, then the number of the different possibilities can be given by the following formula: $C^{4}_{24} \bullet 4! \bullet 8^4 = 10626 \bullet 24 \bullet 4096 \approx 10^9$. This means the order of magnitude is one billion.

TASK 16/ In how many possibile ways can we select 4 square basic elements randomly from the 24 pieces set to get different shapes with a cross-shaped hole in the middle?

Solution: As we would like to assemble four squares, e.g. a configuration with a cross-shaped hole in the middle, the 8^4 factor changes to 2^4, since a square can only be put down in two ways so that the cut-off corner falls in the middle. According to this, the number of the different possibilities reduce the order of magnitude to a million ($\sim 4 \bullet 10^6$), which is – despite the constraints – still a considerable number.

Combinatorial Packaging

TASK 17/ Based on a another idea of the inventor, packaging happens as follows. Each element of the 24-piece packages will be placed randomly above one another and will be covered with trasparent foil. How many different packages are there with each shape using this method?

Solution for Triangles: $6^{24} \bullet 24! \approx 2{,}9 \bullet 10^{43}$
Explanation: a triangle can be set in 6 ways, so 24 triangles above one another in 624 ways. The different sequences of putting the 24 different elements on one another also have to be counted which means the permutation without repetition of the 24 elements, that is 24!

Solution for Squares: $8^{24} \bullet 24! \approx 2{,}9 \bullet 10^{45}$
Explanation: the difference from the triangle is that a square can be set in 8 ways. Other considerations are the same as those written for triangles.

Solution for Circles: $24! \approx 6{,}2 \cdot 10^{23}$

Explanation: in case of the circle we only allow the exact arrangements, which means we count neither axial rotation nor reverse (mirrored) position. So the formula describing the number of the different columns is reduced to the simple permutation without repetition.

SUMMARY: On the basis of the above mentioned we can see that despite the simple set-up of the GAME – three forms and four colours – the number of the combination possibilities is extremely high. It is very unlikely for two toy users to set up the same configuration from the elements accidentally.

In the course of building with the toy we have always stayed within a given form in the previous chapters, but the form and the size of each shape allows for combinations of them. This opportunity results from the main artwork representing the POLY-UNIVERSE, since it can be seen in this first picture that the triangle, the square and the circle can be matched. Considering these facts, we can easily see that merely the closed description of the combination possibilities is an extremely complex task.

Although we took the different solution possibilities into account in this part, we can also note that when playing with the game people usually refer patterns fitting certain rules as the primary object. This is not a constraint dictated by the creator, but a typical human approach to the toy. One of this rules is to aspire to a closed configuration analysed deeper in the previous chapters. It is easy to comprehend that though the set of these ad hoc rules is finite but it is not possible to define them closed.

VI. TRIANGLE: Combinatoric Tasks with rules

Exercises for vertex joints

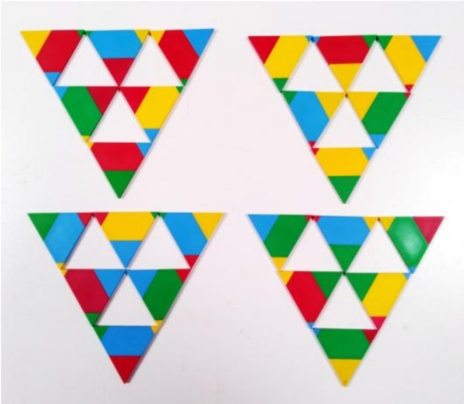
TASK 1/ By joining the vertices, combine three basic triangle elements into a bigger triangle with a triangle in the middle so that smaller identical colours and forms on the vertices also connect. *Solution: 24:3=8*

TASK 2/ Use up all the basic elements.
Master: Is there a solution? If yes, how many different solutions are there? *Solution: It takes about 20 min. With algorithm is possible to calculate the number of different solutions.*

TASK 3/ Once you have found all the triangle formations with a hole, build them together into a bigger formation, so that colours and sizes of the smaller forms on the vertices are identical.

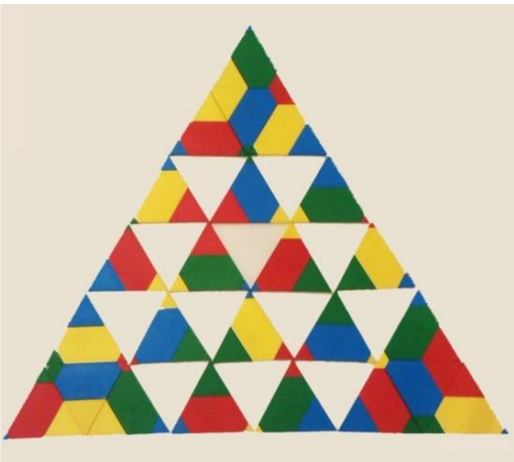
TASK 4/ Using the above rules, create a triangle of 6 elements with three holes in the middle. How many of these can you create by using up all basic elements? *Solution: 24:6=4*

Master: Count in how many different ways you can combine four triangles with holes, so that each of the resulting bigger triangles should be different.
Solution: With an algorithm is possible to calculate the number of different solutions.



TASK 5/ Once you have found all of them, fit them correctly according to colour and size, and build a bigger shape from them.

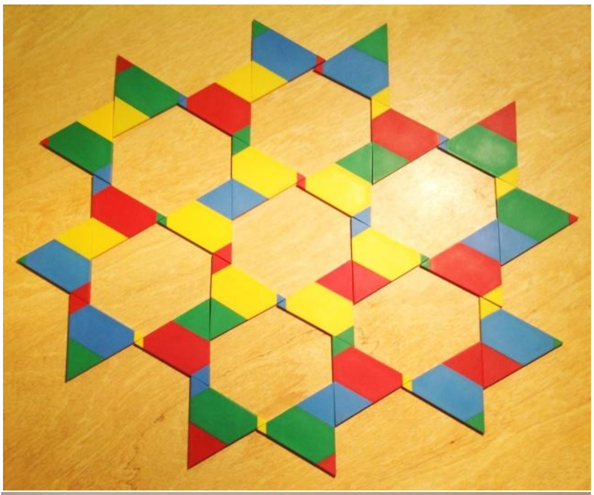
TASK 6/ By using the same rules, put together the largest possible triangle shape by using up all the 24 basic elements. How is this possible?

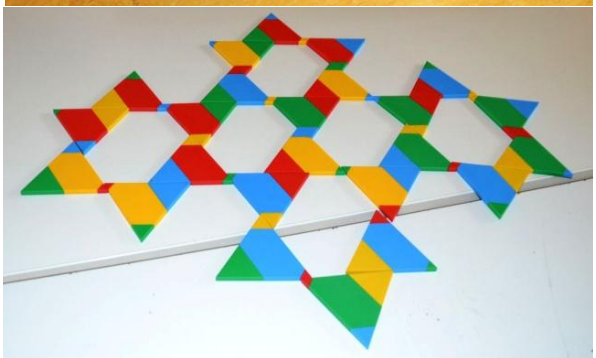


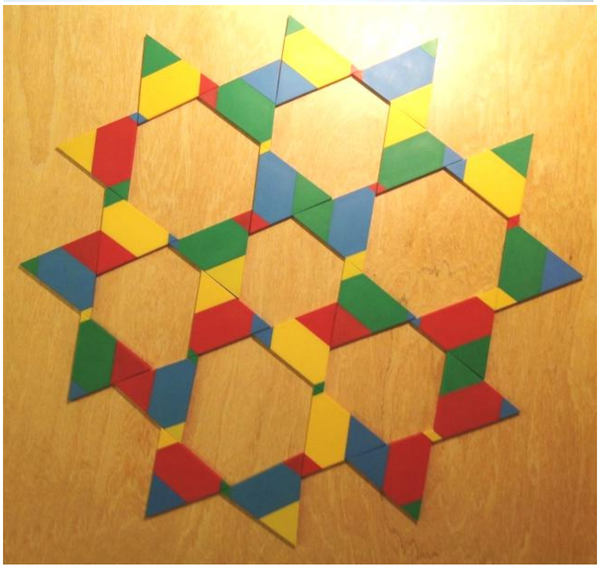
Master: How many different big triangles with holes can you create, so that each of the resulting triangles are different?
Solution: With an algorithm it is possible to calculate the number of different solutions.

Exercises for fitting by sliding

TASK 7/ Master: By using up all the 24 basic elements, put together the largest possible shape. How is this possible to create? Look for the shape with the most combination possibilities and rules.

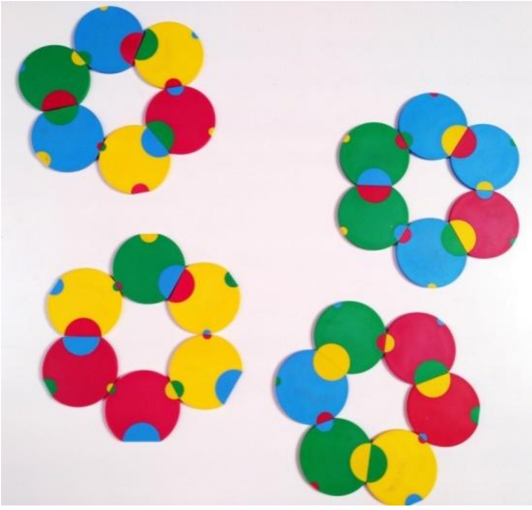






VII. CIRCLE: Combinatoric Tasks with rules

TASK 1/ Take arbitrarily six basic forms from the kit and combine them into a connecting shape (ring) by fitting them according to size. How many shapes like this can you create? *Solution: 24:6=4*



TASK 2/ Take arbitrarily six basic forms from the kit and combine them into a connecting shape (ring) by fitting them according to size and colour. How many shapes like this can you create? *Solution: 24:6=4*
Is there a solution for the case that you do not open the already finished shapes? How many shapes can you create this way? *Solution: 24:6=4*

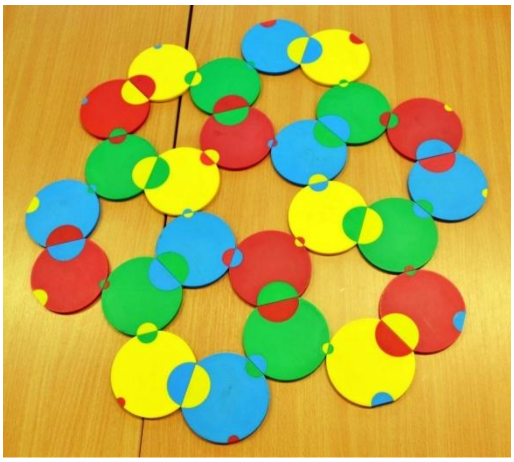
Question: Is it possible to use up all basic elements and arrive at a solution in all cases? *Solution: we tried it in 3-4 cases with success.*
Master: By using the above rules, in how many ways can you create four rings, all with different arrangements?
Solution: With an algorithm it is possible to calculate the number of different solutions.

TASK 3/ Build a connecting shape with correct colour- and size fitting by connecting the rings. How many rings can you connect, with what kind of arrangements? How many possibilities are there to create symmetrical connecting shapes?

Solution: putting together 1, 2, 3, 4, 5, 6 is relatively easy. By using up the entire kit in a symmetrical way you need 7 rings in order to create a connecting formation. It might even take an hour to accomplish it. During 5 years of testing two dozen of results evolved.

TASK 4/ Master: By using the above rules, in how many ways can you put together 7 rings with different connecting arrangements?
Solution: Since you should pay attention to the symmetry of the smaller semicircles connected in the rings, two combination possibilities evolve: arbitrary and symmetrical. Until now we could not estimate the number of possibilities. With an algorithm it is possible to calculate the number of different solutions.

TASK 5/ Recreate the same 7-ring connecting shape with inverse colour connections. More precisely the colour of the basic element and that of the semicircle connecting to it should be identical. Is there a solution?



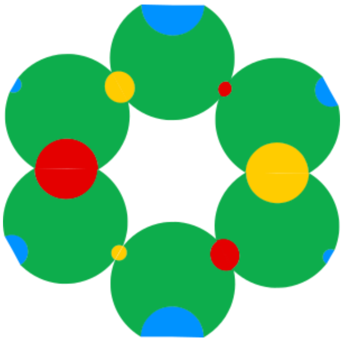
Solution: in a few cases it was possible to do it.

Master: By using the above rules, in how many ways can you create a 7-ring shape with different arrangements? *Solution: With algorithm is possible to calculate the number of different solutions.*

We cannot avoid examining the inner symmetry of ring creation.

First, we choose identically coloured pieces from the kit and arrange them into a connecting shape with correct size and colour fitting (see the graph).

TASK 6/ How many different shapes can we create by using up the entire kit?
Solution: since there are 4 colours and 6 basic elements of each, the number of solutions is 24:6=4, if we disregard the mirror images of shapes.



TASK 7/ How many shapes of different arrangements/symmetries can we create by using up the entire kit? Are there any axially symmetric shapes among them? Are there any of rotational symmetry? Let’s examine symmetry without considering basic colours during fitting, only the sizes.

a) Rotational symmetry in a diagonal setting: identical size semicircles are positioned facing each other and all three sizes are represented in the ring. *Solution: 1 possible arrangement*

b) Rotational symmetry in triangle setting: identical size semicircles are positioned in a triangle setting and two sizes are represented. *Solution: 3 different arrangements; L+M, L+S, M+S.*

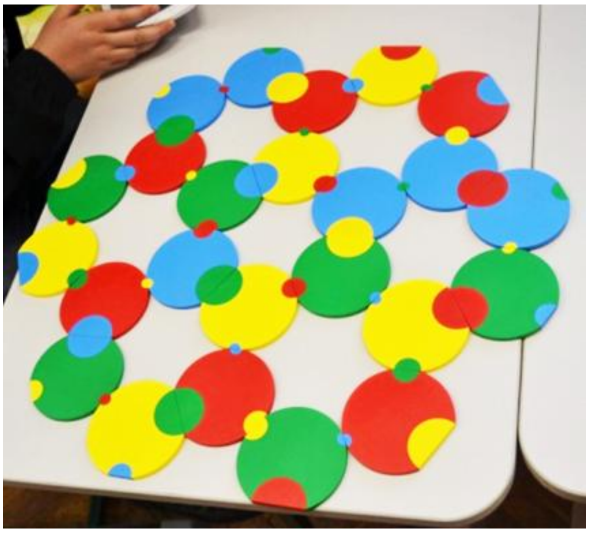
c) Axial or mirror symmetry: all three smaller circle sizes are alternatingly represented. *Solution: 3 different types of arrangements*

d) Another solution involving mirror symmetry: further 6 arrangements are possible, e.g. 3 small, 2 middle, 1 big: 3S+2M+1L; 3S+2L+1M – 3M+2S+1L; 3M+2L+1S – 3L+2S+1M; 3L+2M+1S

Solutions: The number of solutions of inner symmetry examination is 13, meaning that there are 13 ways to begin putting the rings together (in case we only consider the size of the smaller semicircles*).*

TASK 8/Master: In how many different ways can we start putting down the rings, if we consider both the correct size and colour fitting?

TASK 9/ Create the 7-ring shape by involving the different symmetries one after the other. In which case is the exercise solvable? *Solution: rotational symmetry makes it possible in both triangle- and diagonal arrangement.*




TASK 10/ Is the 7-ring shape possible to make, if we don't consider the enlisted symmetries, that is, we join the smaller semicircles arbitrarily to each other? *Solution: yes, most of the solutions are of this kind, the fittings are however not exact.*


Special exercises

TASK 11/ Join three basic circle elements by connecting the vertices of semicircles with respect to size and colour. How many shapes can you connect by using up the entire kit? *Solution: 24:3= 8, because there are 6 large semicircles in each colour, which is enough to create 2-2 shapes.*

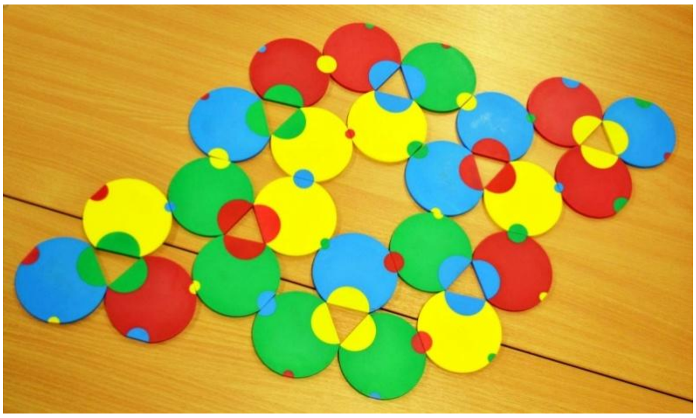
TASK 12/ Join the 8 special shapes together, so that the colours of basic elements stay always different. Is there a solution?

Master: If there is, count how many different 8-shaped arrangements are possible.
Solution: With an algorithm it is possible to calculate the number of different solutions.

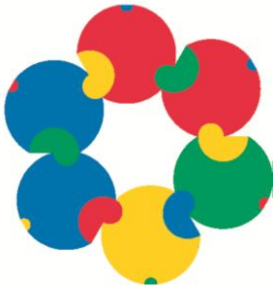
TASK 13/ Join the most possible basic elements by using the rule above, in order to arrive at the most perfectly symmetrical shape. What kind of joining method do you need in order to arrive at a result? *Solution: mixed joining*

Master: How many different arrangements are possible? How many solutions are there?
Solution: With an algorithm it is possible to calculate the number of different solutions.

TASK 14/ Use the entire set and apply the above rules. What kind of solution did you find for the arrangement?



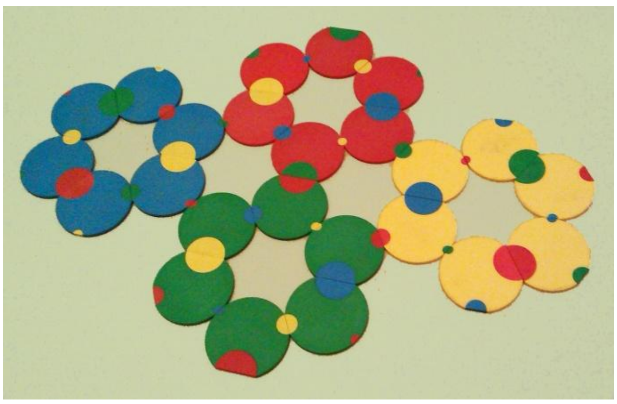
TASK 15/ Join by fitting with sliding 6 basic circle elements so that the identically coloured but differently sized semicircles fit. In how many different ways can you connect the semicircles?
Solution: with rotatinal symmetry e.g. in 3 three different ways – 6xL-M, 6xL-S, 6xM-S. Or e.g. 2xL-M + 4xM-S and its variations. In rotational symmetrical arrangements it is only possible to join them in a mixd (full and sliding) way, e.g. 3xL-M + 3xS-S and its variations….



TASK 16/ How many such shapes can you connect by using up the entire kit, if the colours of basic elements are also identical? *Solution: 24:6= 4*

TASK 17/ In how many different ways can you connect the evolving rings into a cross by using the above symmetry rule? *Solution: always in rotational symmetrical arrangement.*

TASK 18/ Can you join the rings into a cross without fitting by sliding? *Solution: this is not possible; at one point it will not fit.*



TASK 19/ Can you create a 6-ring shape by using up the entire kit, if the basic colours differ from each other (by using mixed arrangement)? *Solution: yes.*

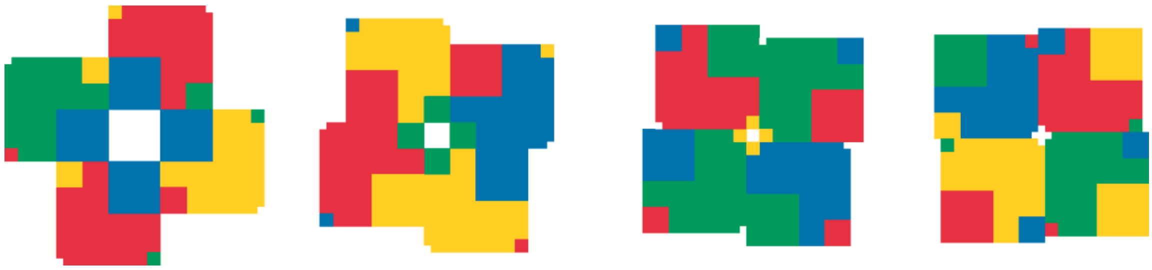
TASK 20/ Master: Can you create a 7-ring shape with fitting by sliding and using up the entire kit, with mixed arrangement? Is this possible with all the different types of inner symmetries?

TASK 21/ Put together all the 24 triangles and circles by using one of the rules. Strive to create the most possible perfect shape and use up all elements. What types of similarities do you identify to nature (e.g. snowflakes)? Look for connections in physics, chemistry, and astrology.

VIII. SQUARE: Combinatoric Tasks with rules

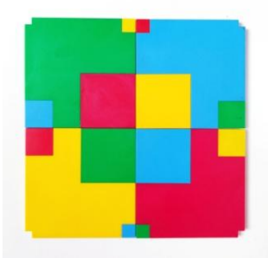
TASK 1/ Take 4 square basic elements and join them into a bigger square, so that the connections in the middle are of identical colour and size. In how many different ways can you connect them? *Solution: 3, that is, by total, sliding and vertex fitting.*

By choosing any of the fitting types, how many solutions are possible by using up the entire kit? *Solution: 24:4=6*
In how many different ways can you launch this exercise? *Solution: 4 colours x 3 sizes + 1 hole = 13 different ways.*

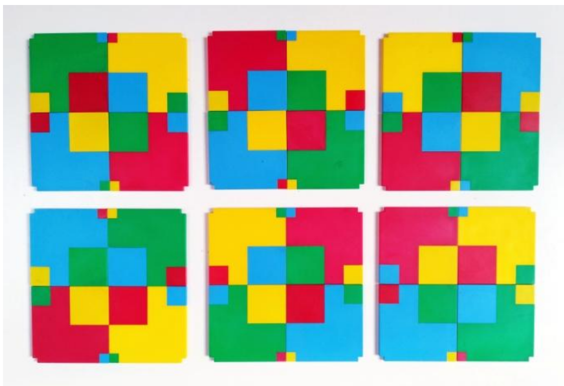


Master: Calculate by colour and by form, how many different shapes of 4 basic forms can you create based on the above rule, so that you always return the basic elements into the kit? *Solution: With an algorithm it is possible to calculate the number of different solutions.*

TASK 2/ Sort the basic elements of the entire kit into fours and combine them in a way that all basic elements and all small, middle and large forms are of a different colour.

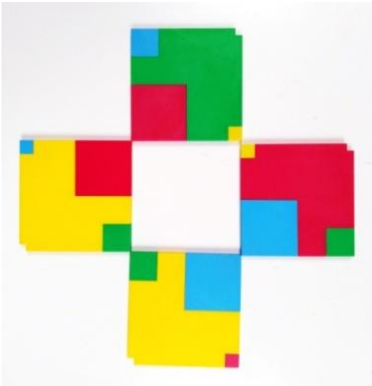


Is there a solution and if yes, how many?
Solution: yes, there is, 24:4=6 by using all elements once.

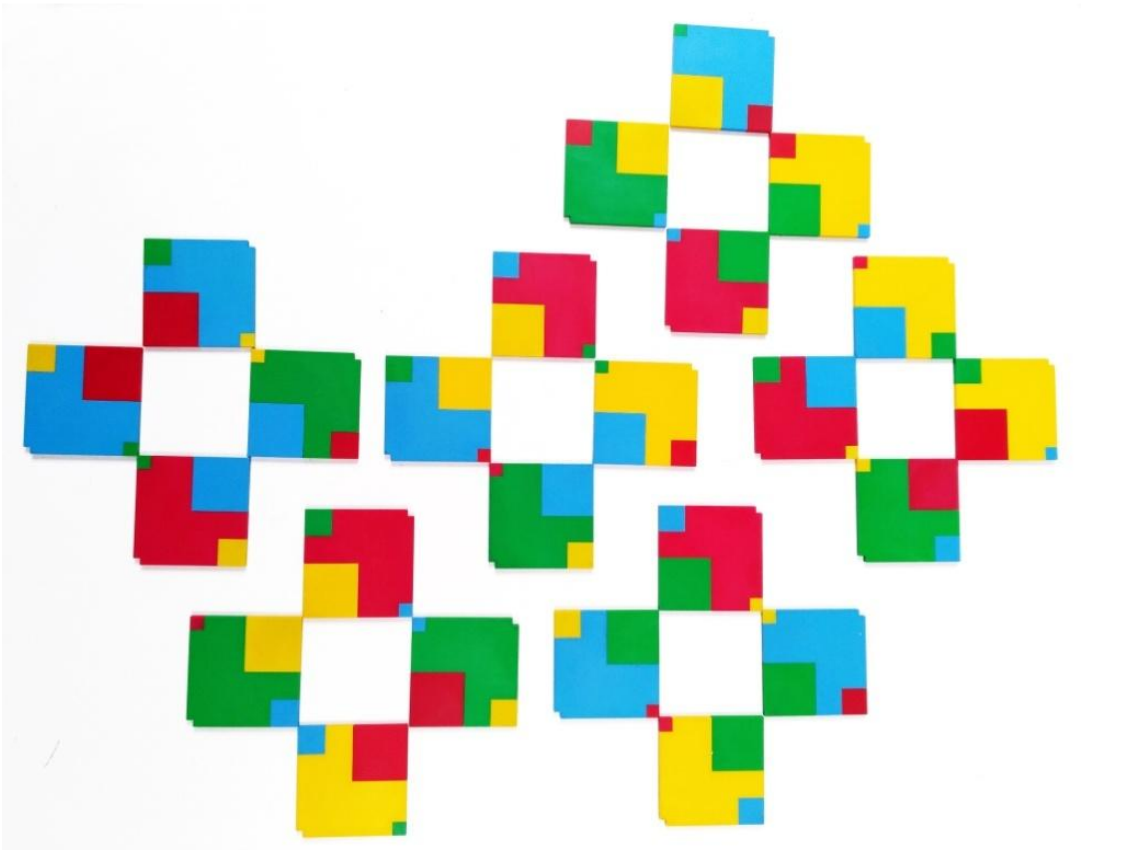


Master: In how many ways can you join the 6 different shapes according to the above rule, by using the entire kit again and again?
Solution: With an algorithm it is possible to calculate the number of different solutions.

TASK 3/ Join 4 square basic elements into a cross, so that they form a hole in the middle. Regularity: the smaller forms should join on their vertices correctly with respect to size and colour. How many can you create from the kit arbitrarily? *Solution: 24:4=6*



Look for as many solutions as possible and use up all elements of the kit. Is there a solution? *Yes.*

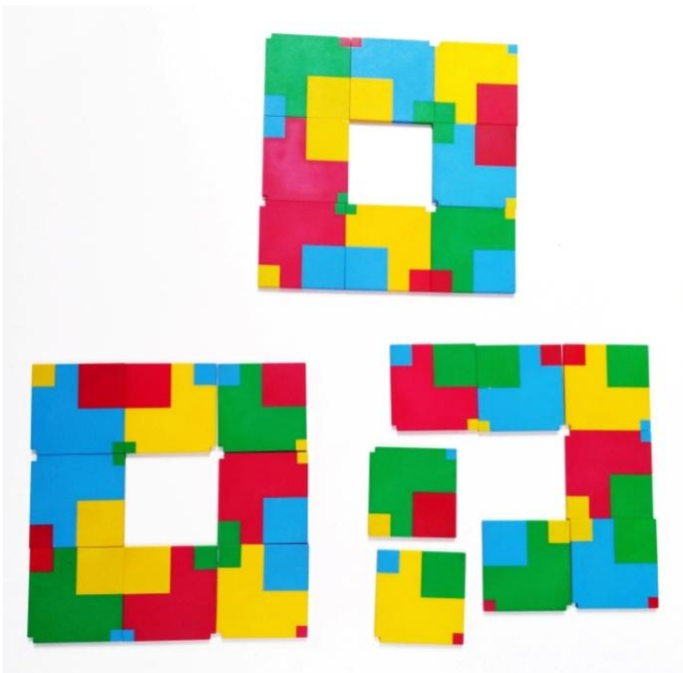


TASK 4/ In how many different ways is it possible to join the vertices correctly with respect to size? *Solution: 4 types of sizes should fit in pairs on the 4 vertices at the same time, while those identically sized might occur twice at one fitting: L+M+S+H*; L+L+M+M; L+L+S+S; L+L+H+H; M+M+S+S; S+S+H+H……..*

Master: If you use only one fitting method at once, is there a solution in all cases for creating 6 crosses with a hole by using up the entire kit?

TASK 5/ Create a bigger square shape in a 3x3 arrangement out of 8 basic elements so that you create a hole in the middle and all vertex fittings are correct with respect to colour and size.

How many different shapes can you create by using up all elements? *Solution: 24:8=3*
Is there a solution for creating all 3 from the kit? *Solution: it was not successful in all three shapes in identical arrangement.* In which fitting is there a solution, though?



Master: If you use all fitting methods, how many solutions do you find while creating the 3 smaller shapes by using up the entire kit?
Solution: With an algorithm it is possible to calculate the number of different solutions.

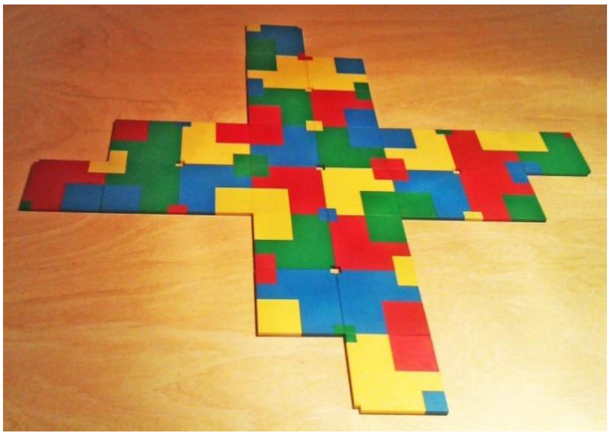
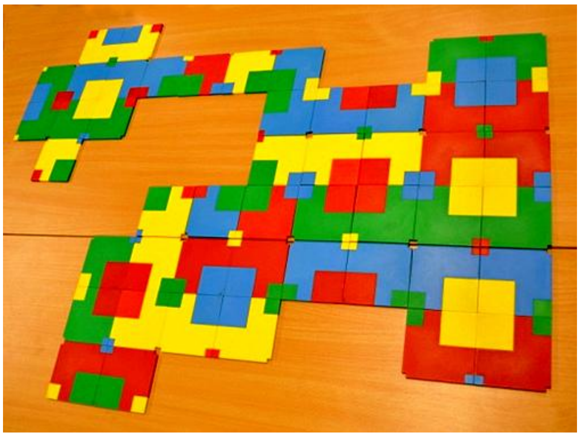
TASK 6/ Create a bigger square arbitrarily with total fitting, in a 5x5 arrangement with a hole in the middle.

Master: How many different solutions are there by using up the entire kit?

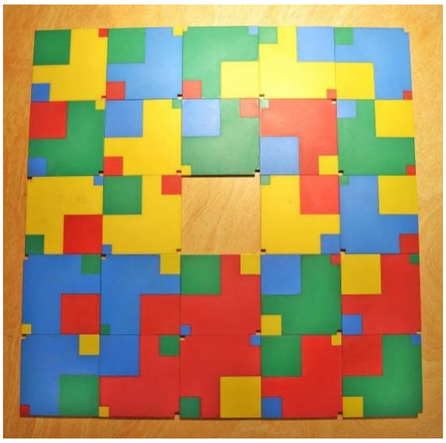
Create the shape with joining the vertices correctly with respect to size!
Master: How many different solutions are there by using up the entire kit?
Solution: With an algorithm it is possible to calculate the number of different solutions.

Create the shape with correct colour fitting at the vertices!
Master: How many different solutions are there by using up the entire kit?
Solution: With an algorithm it is possible to calculate the number of different solutions.

Create the shape with identical size and colour fittings at the vertices. Is there a solution? In which shape can you combine all the elements by this rule?
Solution: there is none. It is only possible in a chain or irregular open shape.

TASK 7/ Create a shape so that both size and colour differ at the vertices! Is there a solution? *Solution: yes, but it is difficult.*



Master: How many different solutions are there according to the above rule and by using up the entire kit?
Solution: With an algorithm it is possible to calculate the number of different solutions.

Master: Is there a solution if we consider the holes in the corners of the basic elements and the colour of the basic element as well?
Solution: With an algorithm it is possible to calculate the number of different solutions.

Exercises with double square kits:

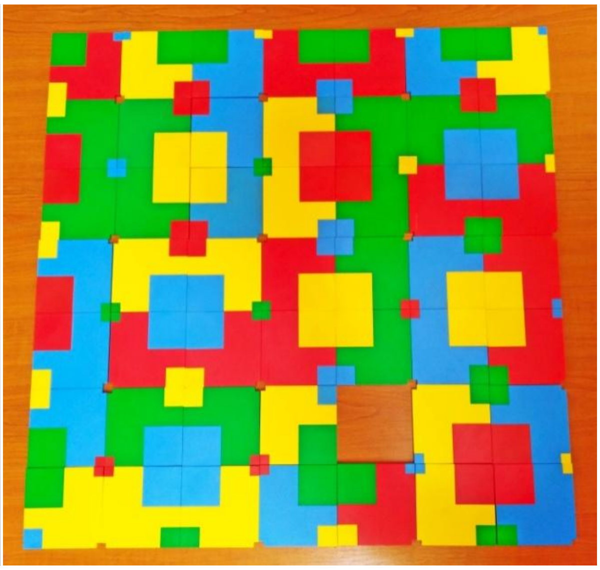
TASK 8/ Create in an arbitrary arrangement a big 7x7 shape with a hole in the middle by using two kits (2x24=48 elements).
Create the shape with correct size fitting at the vertices!

Master: How many different solutions are there by using up the entire kit?
Solution: With an algorithm it is possible to calculate the number of different solutions.

TASK 9/ Create the shape with correct colour fittings at the vertices! Is there a solution? Probably yes.

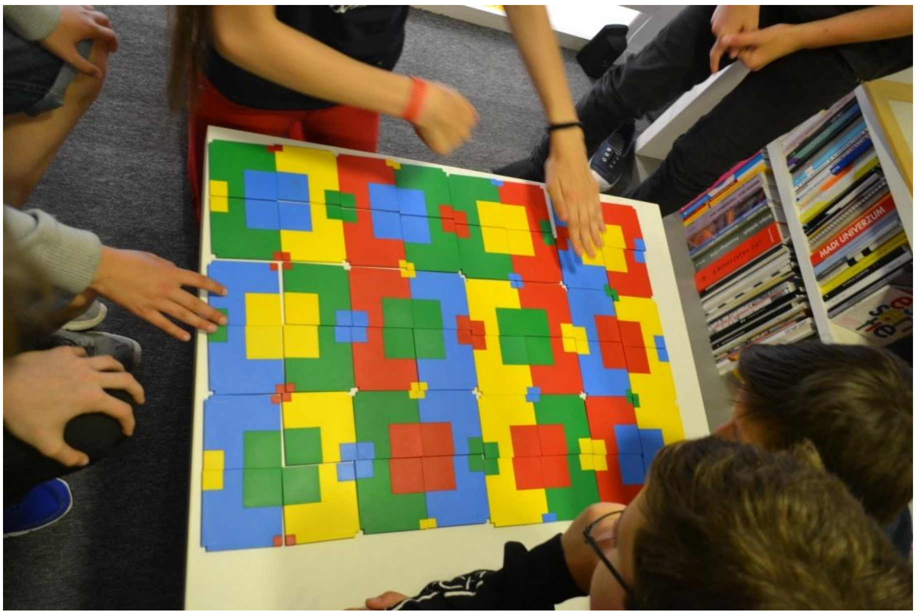
Master: If yes, how many different solutions are there by using up the entire kits?
Solution: With an algorithm it is possible to calculate the number of different solutions.

TASK 10/ Create the 7x7 arrangement with correct size- and colour fittings at the vertices. Is there a solution?

Solution: Up to now we tested this with adults and high school groups and found two solutions involving a hole somewhere else than in the middle. In each case it took several hours for 3-5 persons. With an algorithm it is possible to calculate the number of different solutions.

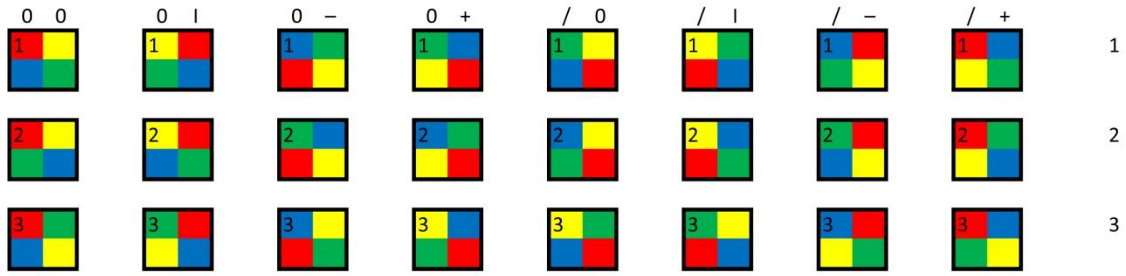
TASK 11/ *If you find it difficult to give a solution, experiment with a 6x8=48 closed square shape!*



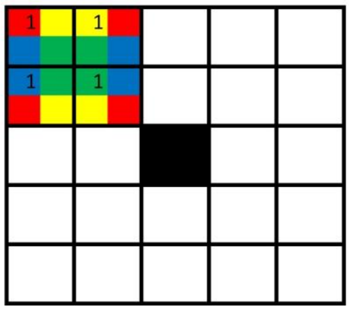
Master: How many different solutions are there by using this rule and involving both kits?
Solution, suggestion: About a dozen of solutions were born until now, an algorithm would be necessary to know more.

IX. ALGORITHM Conception

We examined together with a group of professionals the number of possible solutions for creating regular, closed formations from whole sets of Poly-Universe squares with respect to specific rules. This preparation work, on the one hand, opens a new perspective for educational methodology; on the other hand, it secures a basic concept for further research in mathematical possibilities. We introduce some of the approaches in the supplement.



A fenti négyzet-sorozat a piros, sárga, zöld és kék összes lehetséges színkombinációját tartalmazza. **Feladat:** töltse ki a jobb oldali négyzetet úgy, hogy minden sarkon négy azonos szín találkozzon! A fenti elemek többször is felhasználhatóak, de az azonos sorban lévők összesen legfeljebb nyolcszor szerepelhetnek!



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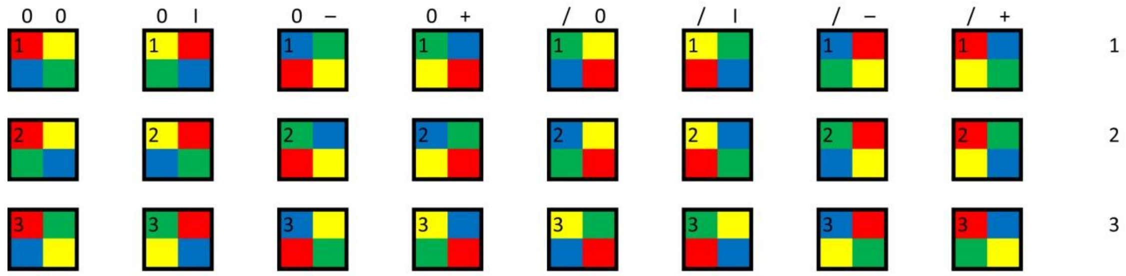
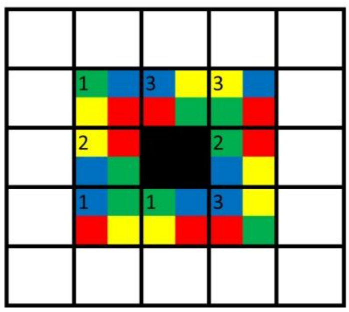
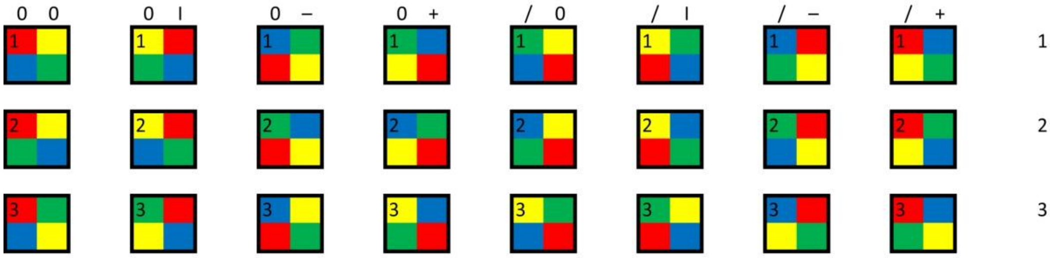
0 db 2-es
0 db 3-as



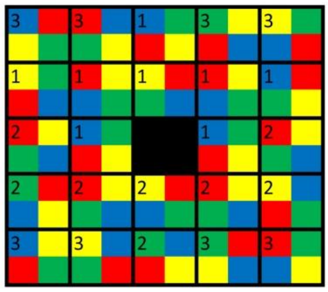
A fenti négyzet-sorozat a piros, sárga, zöld és kék összes lehetséges színkombinációját tartalmazza. **Feladat:** töltse ki a jobb oldali négyzetet úgy, hogy minden sarkon négy azonos szín találkozzon! A fenti elemek többször is felhasználhatóak, de az azonos sorban lévők összesen legfeljebb nyolcszor szerepelhetnek!



3 db 1-es
2 db 2-es
3 db 3-as



A fenti négyzet-sorozat a piros, sárga, zöld és kék összes lehetséges színkombinációját tartalmazza. **Feladat:** töltse ki a jobb oldali négyzetet úgy, hogy minden sarkon három azonos szín találkozzon! A fenti elemek többször is felhasználhatóak, de az azonos sorban lévők összesen legfeljebb nyolcszor szerepelhetnek!



8 db 1-es
8 db 2-es
8 db 3-as

Lépések:

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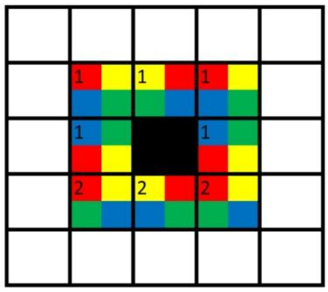

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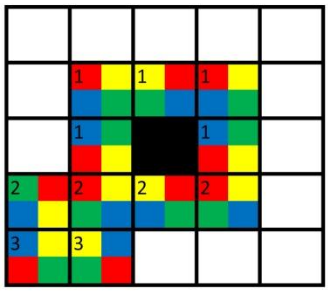

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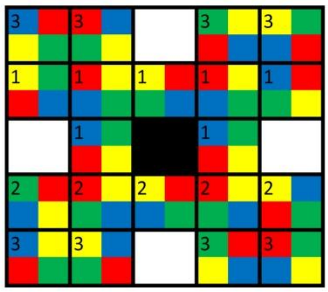

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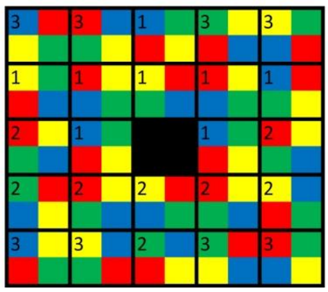
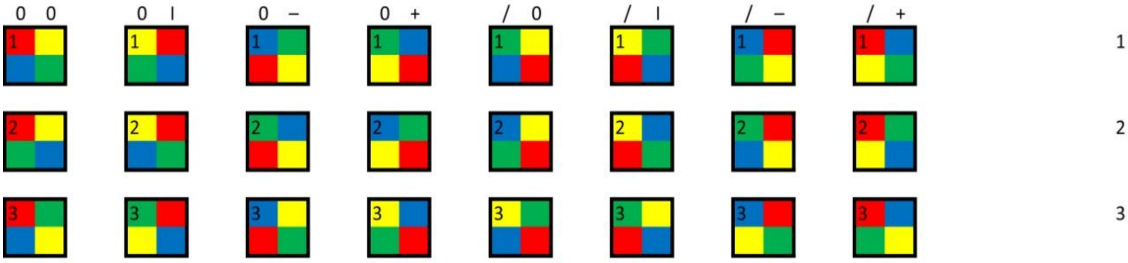
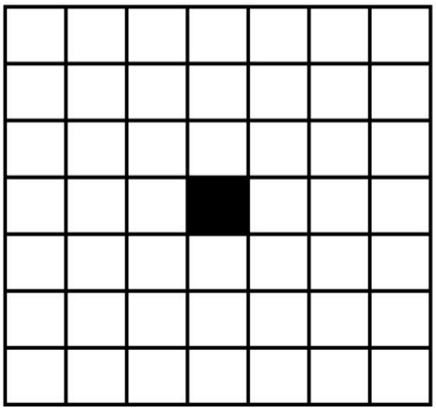
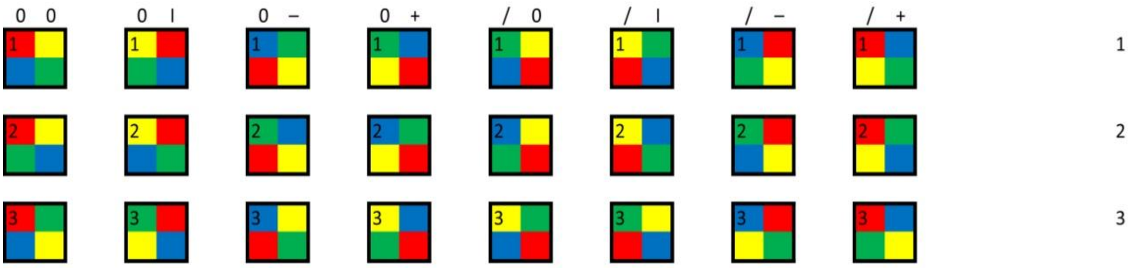




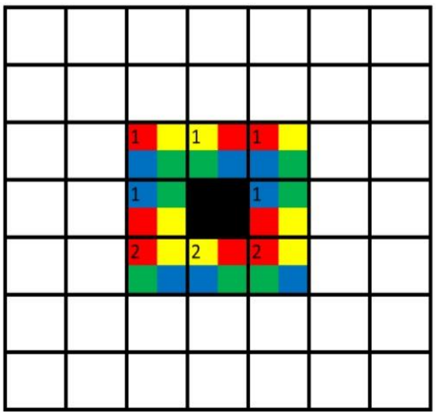
A fenti négyzet-sorozat a piros, sárga, zöld és kék összes lehetséges színkombinációját tartalmazza. **Feladat:** töltse ki a jobb oldali négyzetet úgy, hogy minden sarkon négy azonos szín találkozzon! A fenti elemek többször is felhasználhatóak, de az azonos sorban lévők összesen legfeljebb tizenhatszor szerepelhetnek!



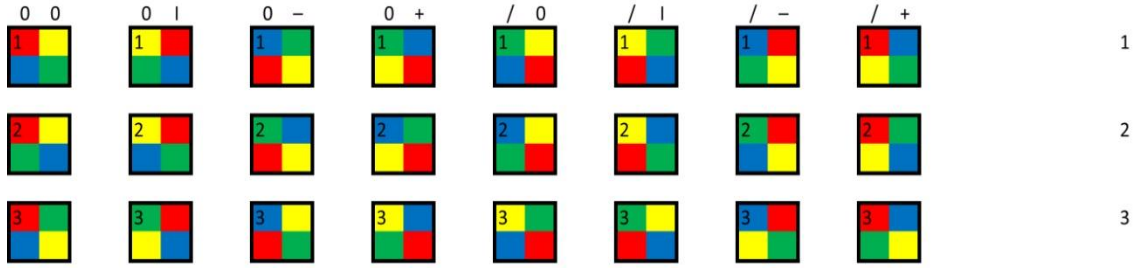
0 db 1-es
0 db 2-es
0 db 3-as



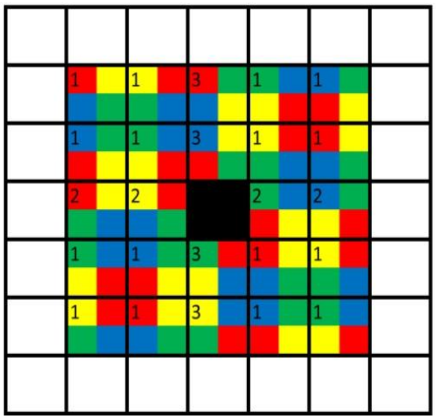
A fenti négyzet-sorozat a piros, sárga, zöld és kék összes lehetséges színkombinációját tartalmazza. **Feladat:** töltse ki a jobb oldali négyzetet úgy, hogy minden sarkon négy azonos szín találkozzon! A fenti elemek többször is felhasználhatóak, de az azonos sorban lévők összesen legfeljebb tizenhatszor szerepelhetnek!



5 db 1-es
3 db 2-es
0 db 3-as



A fenti négyzet-sorozat a piros, sárga, zöld és kék összes lehetséges színkombinációját tartalmazza. **Feladat:** töltse ki a jobb oldali négyzetet úgy, hogy minden sarkon négy azonos szín találkozzon! A fenti elemek többször is felhasználhatóak, de az azonos sorban lévők összesen legfeljebb tizenhatszor szerepelhetnek!



16 db 1-es
4 db 2-es
4 db 3-as

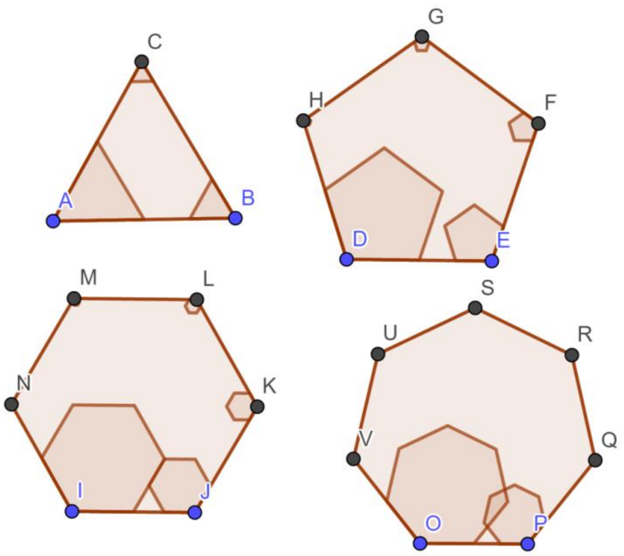
X. GEOGEBRA in Poly-Universe

Nowadays one can be most effective in education by using computer methods as well apart from manipulatives.

The Poly-Universe applications developed for laptop and mobile enable to engage with the same exercises and problem-solving tasks as the tool itself. Applications are practical to use when real sets are not available, when travelling etc.

In the following section we use GeoGebra in order to further consider ideas raised by the Poly-Universe and to construct the elements.

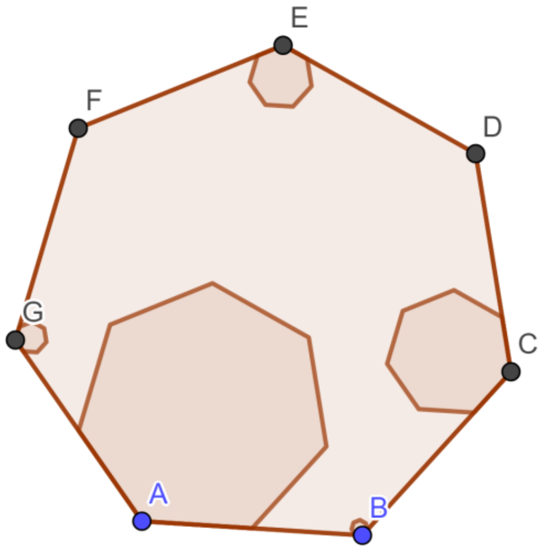
1. Let's consider what would happen if we cut (colour) the respective polygons on the consecutive vertices of pentagons, hexagons etc, instead of triangles or squares, by using ½ similarities. On the shape below we see the mentioned construction in the case of a pentagon, hexagon and heptagon as well. And immediately we face a surprising quality. In the case of a hexagon, the biggest hexagon created at two consecutive vertices touch each other on one-one side, in the case of the heptagon they even reach into the area of the other. Probably we did not think of this before constructing it. And immediately there rises a new question. Is it possible to arrange the polygons at the vertices in such an order that they will not intersect? With any polygon?



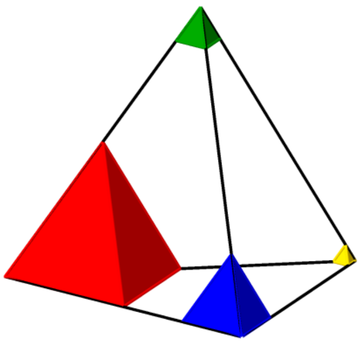
The figure shows a possible arrangement in the case of a heptagon.

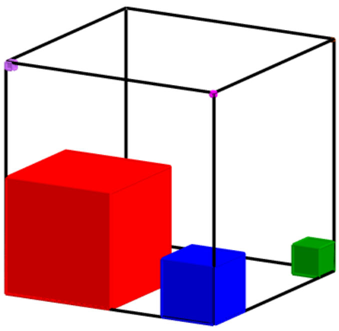
If we consider the area of the original polygon as the unit, what is the sum area of the small (coloured) polygons at the vertices? Calculate 3, 4, 5 etc. for polygons. The area sums form a series. Is this series convergent? If yes, find the sum of the series!

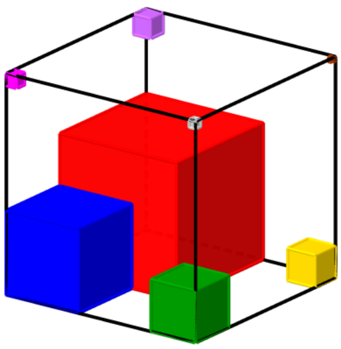
The same questions can be considered for the specific perimeters as well.



2. Another further consideration is the increase of dimension number. In the case of the tetrahedron and cube, the shapes can be easily constructed in GeoGebra.





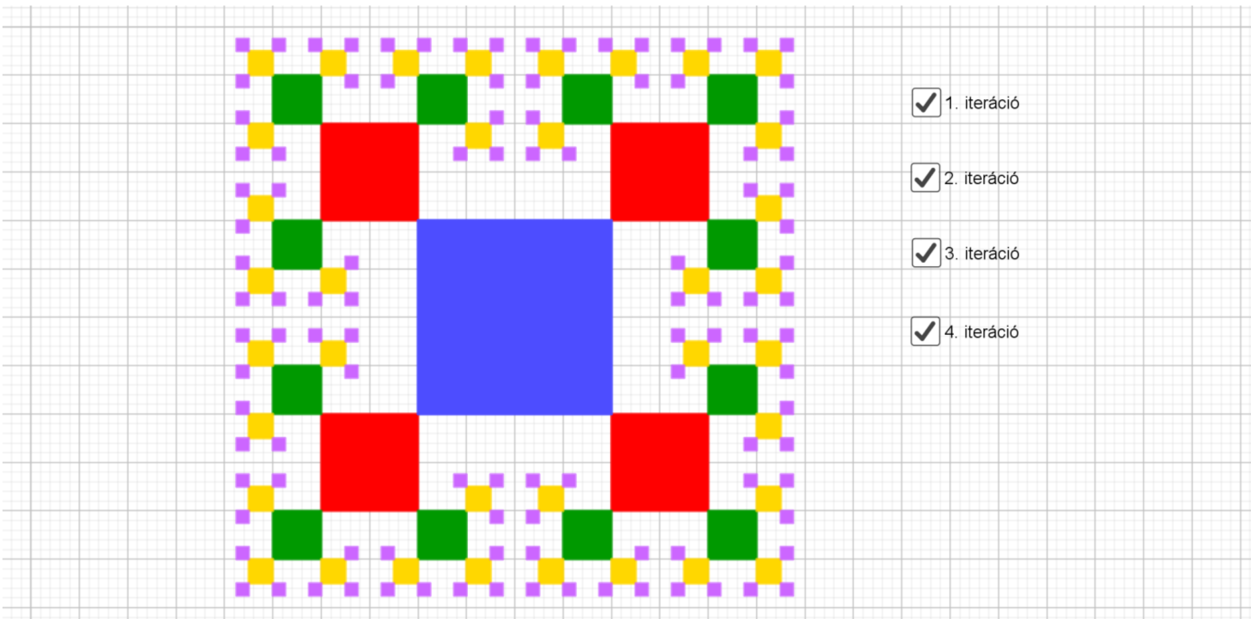


In the case of the cube, if we consequently apply the ½ similarity ratio, the small cube created at the vertex in the 4th step will be almost invisible. Here comes the next question. Which is the largest similarity ratio which avoids the intersection of the small cubes in the vertices?

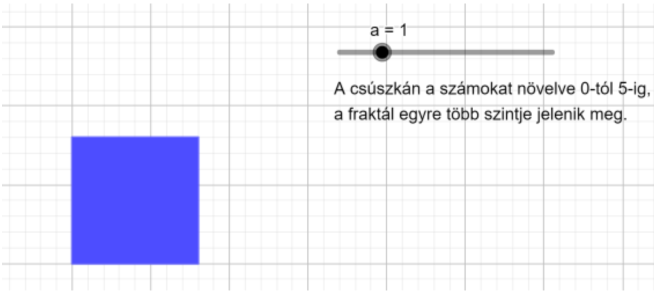
We can justify by calculation that this ratio is exactly the golden ratio: $\varphi = \frac{1+\sqrt{5}}{2}$.

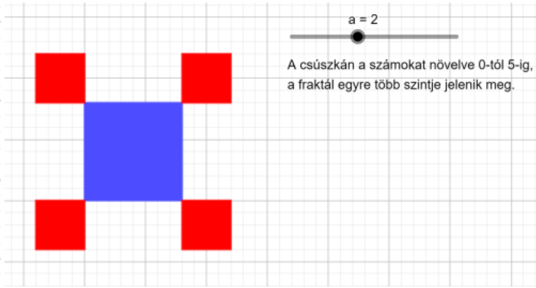
The above figure on the left shows the „cube series” constructed with the ½ similarity ratio, while the one on the right with golden ratio. The constructions can be accomplished easily and quickly with the “dilate from point” command. GeoGebra also offers the opportunity to enjoy 3D graphs with space vision glasses, see the figure below.

3. Using the dynamic possibilities of GeoGebra, it is possible to construct fractal figures inspired by Poly-Universe. On the first figure we show the different iterational steps of the fractal using checkboxes.

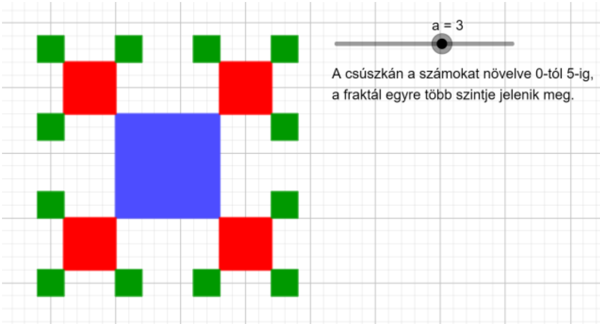


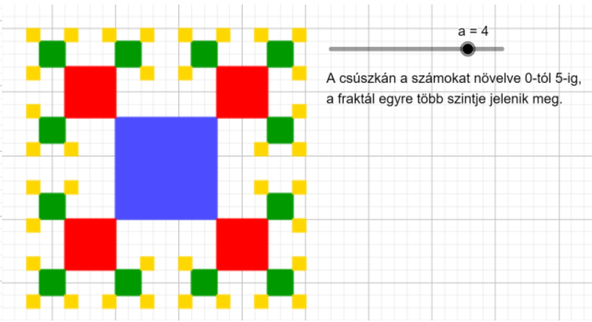
Even better is the illustration using the slider. The figure below shows the phases belonging to the different values of the slider.

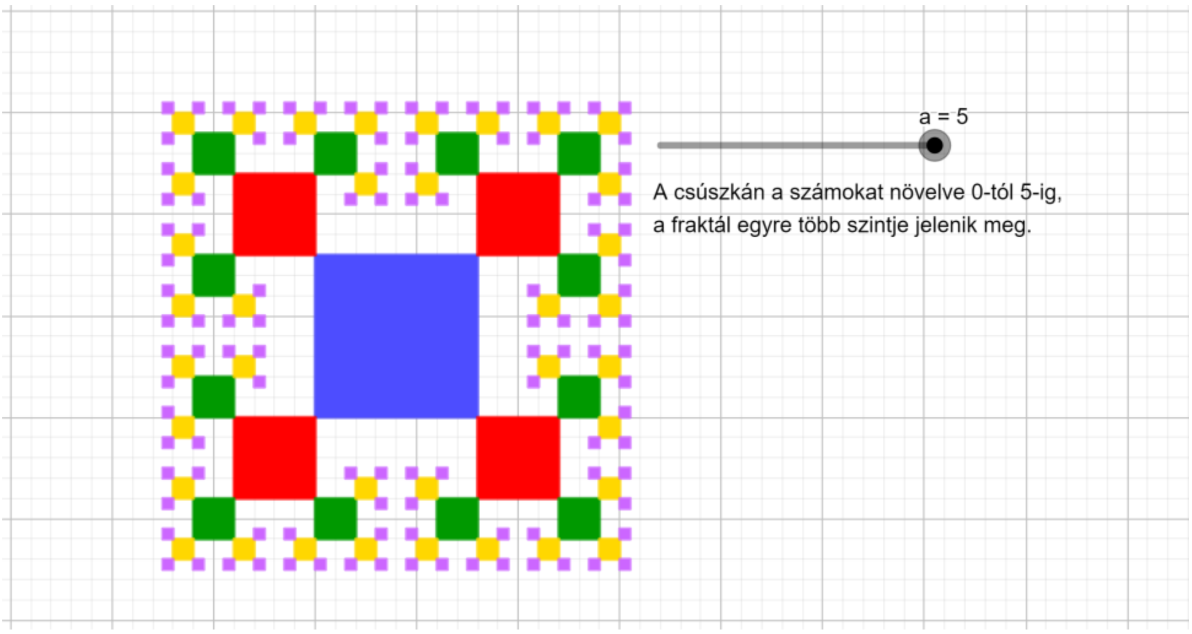




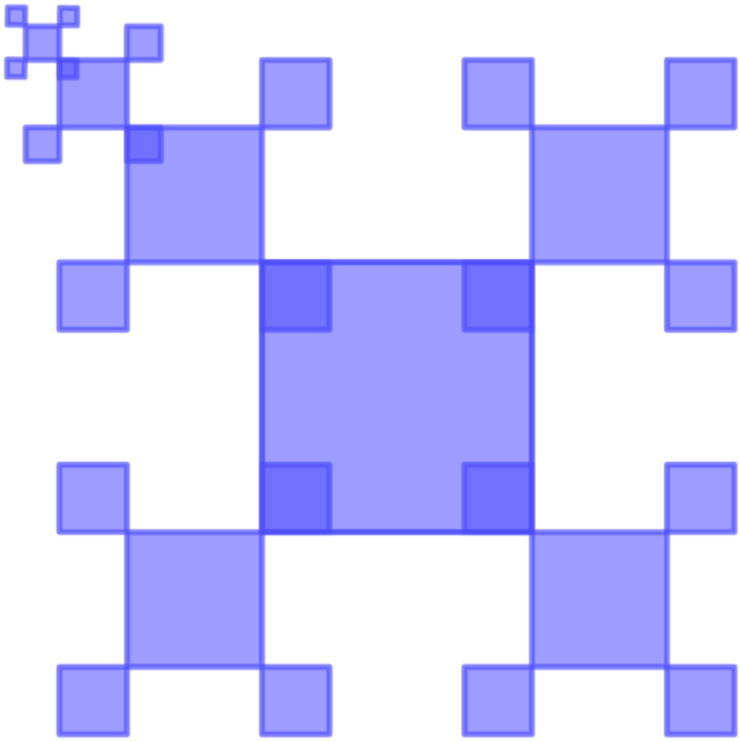
By increasing the values on the slider from 0 to 5, increasingly more levels of the fractal appear.





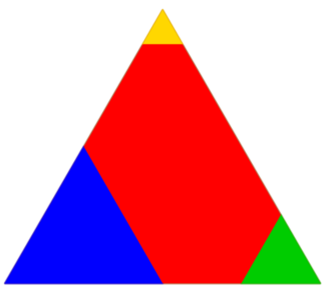


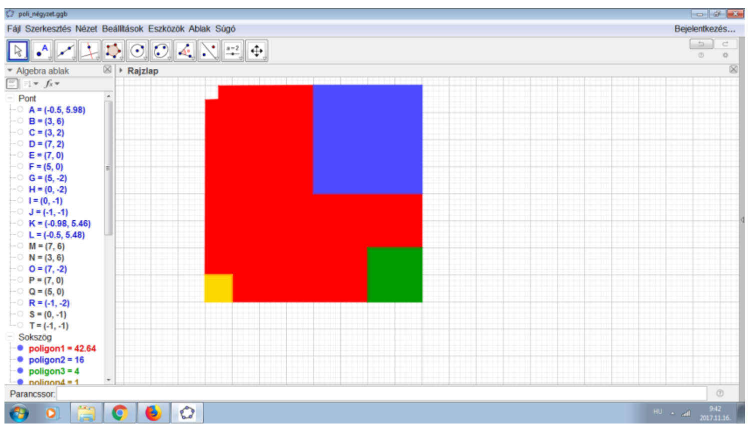
We can even create a tool to make construction easier. If we click on the neighbouring vertices of a square, it draws 4 new squares around the original square. The sides are parallel to the original square's sides, the length of the sides is half of the original square's sides.



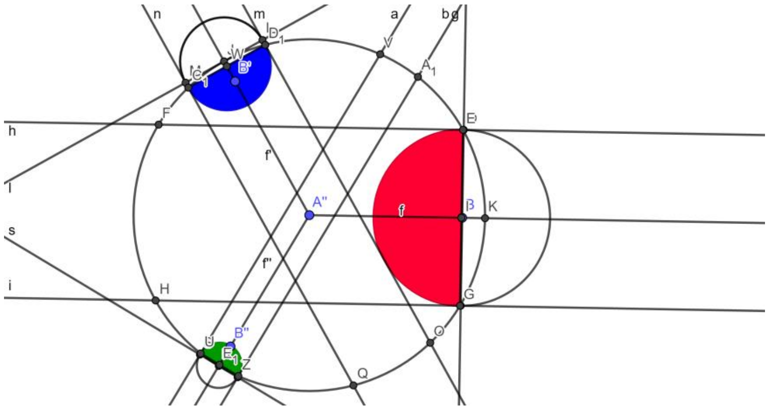
4. Constructing the Poly-Universe basic elements in GeoGebra.

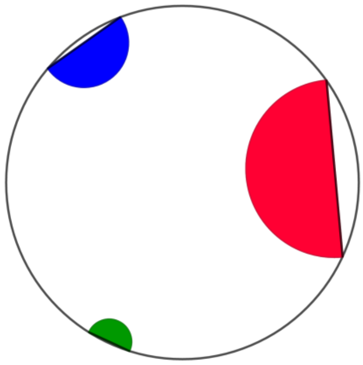
   To achieve a more exact construction in the case of the square, we suggest turning on the grid. Both in the case of the triangle and the square, the appropriate element can be constructed most quickly and simply by the “dilate from point” command.





The construction of the circle element, however, is significantly more complicated. The picture on the left shows the added lines as well, the one on the right is the ready image only.





Summary: The construction tasks listed above can be linked to upper grade and secondary school mathematics materials. Triangle and square constructions can be done in elementary school upper grades, the construction of the circle, however, is a challenge even in secondary school. The area and perimeter calculations of the 1st exercise with a given number of sides can be connected to elementary and secondary school teaching (perimeter, area of similar plane figures). By heavily increasing the number of sides, the convergence of the series sums of perimeter and area belongs already to the university curriculum. The knowledge material in the intersection of mathematics and arts, usually missing from school education, is represented here by the golden ratio and fractals.

We use the dynamic opportunities of GeoGebra (checkbox, slider) and even the transformational approach is strongly present (120 degree turns in the construction of the circle) and the central similarity in the construction of the two other elements and their spatial extensions.

PUSE Study

on the use of the Poly-Universe game family

Budapest Fazekas Mihály Practicing Primary School and Grammar School

Report by

Jakucs Erika
Jencsikné Végh Adrienn
Kocsis Szilveszter
Pereczes Marianna
Szigeti Rita

CONTENTS:

I. CHARACTERISTICS OF OUR SCHOOL, LOCAL CHARACTERISTICS

Our school is located in the 8th district of Budapest. The school building and the education in the school has long and rich history. It was inaugurated in 1911’s summer. In 1960 it was changed to have 12 grades and renamed Budapest Fazekas Mihály Practicing Primary School and Grammar School. Since 2017 the school has been functioning as a “base school” which means it is one of the best providers of education in Hungary. Since 2018 it has been also functioning as a Mathematical centre. Our teachers and students are proud of their school because of its education and also because the school building is an outstanding architectural value designed by Lóránd Balogh.

The structure of the school:

- Primary school: 1-8. grades (17 classes)
- Six-grade grammar school: 7-12. grades (12 classes)
- Four-grade grammar school: 9-12. grades (8 classes)

Special functions of the school

- The school is an advanced matura examination centre of Biology, Physics and English language.
- Metropolitan and national competitions are organized every year.
- Talent care extracurricular lessons are provided for our students every year (based on the students’ needs and the possibilities).
- We organize conferences and methodology presentations.

Our students

The enrolment area of the primary school is Budapest and the agglomeration. The high school attracts students from the whole country of Hungary. Fazekas can be a good choice for those students who like studying and for those for whom the fact that being a practising school involving lot of guests, demonstration lessons and frequent changes of activities does not cause difficulties. The primary school for 1-4-graders does not have any special classes.

The six-grade grammar school starts with two parallel classes, with a general curriculum and a special mathematics section class.

The four-grade grammar school also starts with two parallel classes: social sciences and natural science.

The experience of recent years and decades shows that the majority of children attending our school come from an intellectual family where learning and knowledge have been valuable for generations. There are many students whose parents or grandparents attended Fazekas. Of course, this does not mean that the school is prevented from general social problems like divorce, learning disorder, etc. We try to fix these problems with our own tools for e.g. developer pedagogical work, speech therapist's help to catch-up, differentiation according to the students' skills, and a lot of patience and empathy.

Grades 1-4.

The 1-4-grader teachers' goal is to give demonstration lessons, consultations and accredited courses in which as a training school we support teacher trainings and self-education for both teachers working in the capital and in the countryside. Our courses are often attended by teachers from abroad. We have returning guests from England and Finland. Many of the school's teachers are authors or co-authors of textbooks and methodological materials. Our teachers are often invited to hold methodological presentations which shows their rich experience. Our talent care extracurricular lessons are visited by students from each district of the capital.

Our day care educators also hold courses about day care learning methods and about how students could usefully spend their leisure time.

In conclusion, there are two outstanding activities at the grades 1-4. department: talent care and professional services. Every student studying in Hungary can join the talent care extracurricular courses independently of their finances, cultural background, or which school they are attending.

Grades 5-9. and the grammar school

The work of grades 5-9 and the grammar school is only partially separated. Most of our teachers teach in both parts. For this reason our specialisations are similar as well. Our extracurricular Mathematics classes are open for everyone, so many students from the capital and the agglomeration visit them from each grade.
We have two very special extracurricular Mathematics courses: the "super course" and the course taught by Sándor Dobos which is a Maths competition training course.

Regular meetings are organised by us for the mathematical work communities in Budapest, and we are also responsible for the mathematical competitions for the primary schools in the capital.

What is important to mention:

The international reputation of our school is primarily due to its mathematical talent management. In addition to our special mathematics section, heuristic education is provided in all classes. Our cabinets are rich of varieties of devices for discovering, and we really use these tools for our lessons every day. Therefore, the deployment of a new tool among our students does not necessarily have a demonstrable effect, while in schools where the use of devices is less typical, the impact may be more visible. This feature must be taken into consideration when processing the results.

II. THE PROJECT: GRADES 1-4

Preparation

We started using the sets in January 2018. The chosen pupils were first graders because the teachers who participated in the project were teaching in first grade.

In January, we made the input measurement before we employed the students to use the sets. We primarily measured the level of attention and concentration. We measured two groups of 16 which had been studying Mathematics based on the same methodology using the same book. Data of the input measurement was recorded in an Excel table and sent to the PUSE Team psychologists for evaluation.

We started using the game on 2 February 2018, according to the concept developed in November after we consulted with the psychologists when the input measurements had been done and evaluated. To use the game, we tried to fit into the original progress of the Mathematic lessons and also easily insert it into the curriculum. Therefore, we took the 1-4 grade Mathematics topics as the basis. The topics are concentric, which means that the topics of the first class are repeated in the higher grades, giving the children deeper and deeper experience and knowledge.

Tasks were assigned to topics, indicating which grade they were recommended for based on their difficulty. We chose those ones that can be clearly linked to the Poliuniverzum game family having a developmental effect.

Topics:

I. Diagram logic
II. Functions, sequences
III. Geometric experience
IV. Arithmetic, algebra
V. Combinatory

We had classes once a week in the morning at the afternoon. The lessons took 45 minutes. The kids could play either at the Math classes on one week or in the afternoon club the other week. We measured the 16 students' achievement levels at the beginning and at the end of the programme and 25 students at the lessons themselves since the primary aim of the game is to cooperate with the whole class.

Playing the game and the experiences

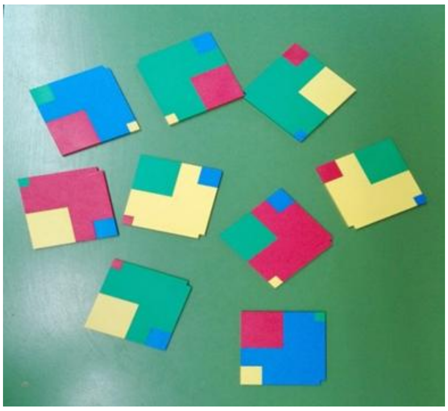
The first lesson (2 February 2018)

Diagram logic – classification (devision)

Aim: creating rules – feature description, decision-making about the pieces – if they have the given properties, negotiation, coding, creativity, improvement of the aesthetic feel, formation of thoughts via statements and decisions

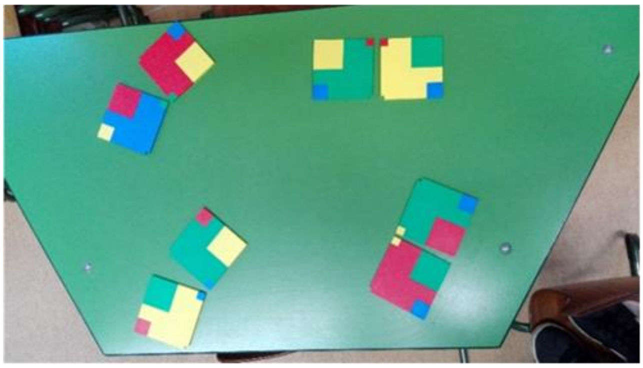
Deciding about the truthfulness of the statements (grades 1-4.)

Groups of 4 received a square package. They observed the pieces, collected statements by finishing the following sentences: There is a …, There isn't a …, It is true for all of them…, It isn't true for any of them…,



Selection based on given aspects (grades 1-2.)

The kids continued a selection recognising the rules. Rule: based on the colour of the smallest piece there were 4 selection processes (red, yellow, blue, green)



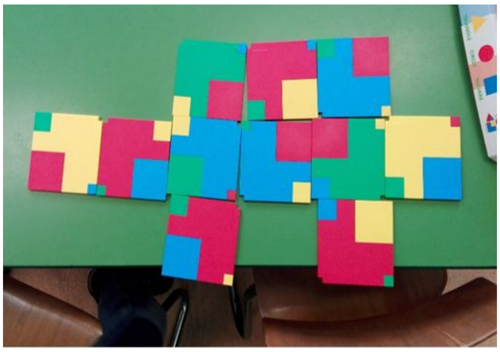
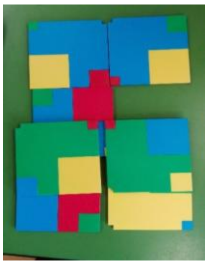
Selection based on personal preferences (grades 1-2.)

The kids chose the selection rule in groups. Then they continued each other's selection, e.g. they selected based on the colour of the majority of the pieces; they divided the pieces into

4 groups based on the colours of the square elements or they divided the pieces into 4 groups based the colours of the medium-sized squares.

Building plane figures (getting geometry experience, grades 1-4.)

The kids built freely, without any given aspects.

Our experience: The kids were enthusiastic when they were using the sets. Working in groups of four was not a problem for them. They cooperated with each other, didn't argue. They enjoyed using the sets freely the best.

They learned the following mathematical concepts by using the package: a square, not a square, connection point, identical, as big as, angles, sides, mosaic.

They got experience in comparisons, observations and making pictures.

The second lesson: 9 February 2018

Aim: creating rules – feature description, decision-making about the pieces – if they have the given properties, negotiation, coding, formation of thoughts via statements and decisions

Deciding about the truthfulnes of the statements (grades 1-4)

The kids were working in groups of 4 with a triangle set. They observed the pieces, collected statements by finishing the following sentences: There is a …, There isn't a …, It is true for all of them…, It isn't true for any of them…,

Selection based on given aspects (grades 1-2.)

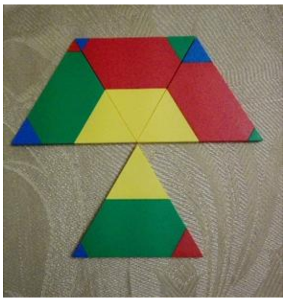
The kids continued a selection recognising the rules. Rule: based on the colour of the biggest part of the piece there were 4 selection processes (red, yellow, blue, green).

Selection based on personal preferences (grades 1-2.)

The kids chose the selection rule in groups. Then they continued each other's selection, e.g. there were 4 selection processes based on the colour of base form, there were other 4 selection processes based on the colour of the medium size triangle. They found other aspects and continued selecting.

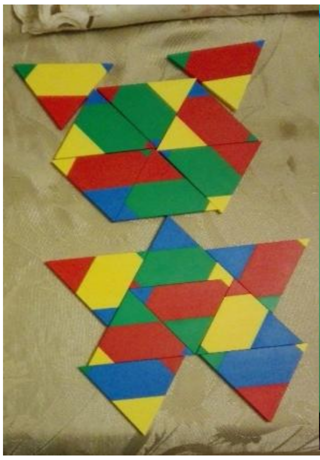
Building plane shapes (getting experience in geometry, grades 1-4)

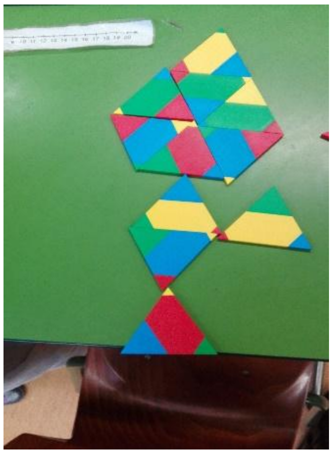
They could build freely using the triangle package without any given aspects.



They got experience in comparisons, observations and making pictures.

We used the package as a **tangram**. They built different shapes in groups e.g. a teddy bear, a ship, a flower etc.





Our experience: Using the package as a tangram was very popular, they were very enthusiastic while they were cooperating with each other. Their pictures from the shapes were beautiful.

They learnt the following concepts by using the package: a triangle, not a triangle, connection point, identical, as big as, angles, sides, and tangram.

The third lesson (16 February 2018)

Diagram logic – classification (division)

Aim: creating rules – feature description, decision-making about the pieces – if they have the given properties, negotiation, coding, formation of thoughts via statements and decisions

Deciding about the truthfulness of the statements (grades 1-4)

Groups of 4 received a circle package. They observed the pieces, collected statements by finishing the following sentences: There is a …, There isn't a …, It is true for all of them…, It isn't true for any of them…,

Selection based on personal preferences (grades 1-2)

The kids chose the selection rule in groups. Then they continued each other's selection. They selected based on the colours of large, the medium and the small semicircles. There were 4 selection processes in each size (red, yellow, green and blue).

Selection based on given aspects (grades 1-2.)

The kids continued a selection recognising the rules. Rule: based on the colour of the biggest part of the circle there were 4 selection processes (red, yellow, blue, green).

Building plane shapes (getting experience in geometry, grades 1-4.)

They could build freely using the circle package without any given aspects.

Our experience: they cooperated with each other enthusiastically. They created exciting mosaics.

They learnt the following mathematical concepts: a circle, not a circle, semicircle, curve, arc, angle, connecting point, identical, as big as, side.

They got experience in comparisons, observations and making pictures.

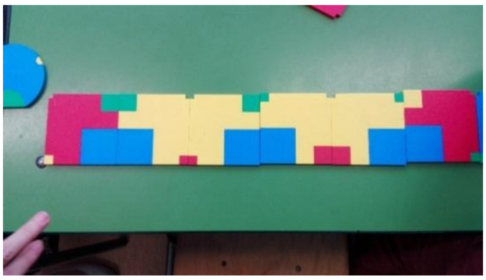
The fourth lesson (23 February 2018)

Functions, sequences

Aim: developing the ability of recognising relationship, feature description, decision-making about the pieces – if they have the given properties.

Sequences based on given aspects (grades 1-4)

The kids continued the sequence using the **square package.** Rule: the connected pieces in the sequence must be the same coloured and the sized squares.



They made their own sequences freely, based on the rules which they created themselves: E.g. the sequence was repeated in periods. The colour of the large square was red, blue, yellow and green.

Building plane shapes (getting geometry experience, grades 1-4.)

They created mosaics based on given aspects. Rule: the connecting points had to be identical in colour and size.

They learnt the following concepts: repetition, periodic, sequence.

They got experience in comparison, observation, constructing and periodic repetition.

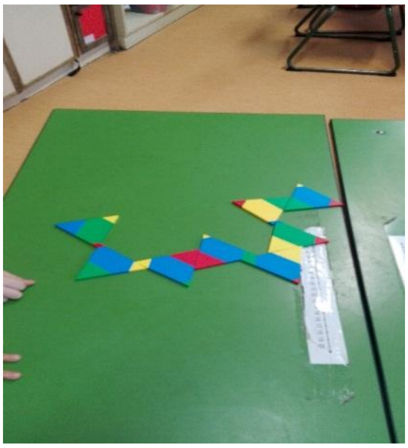
The fifth lesson (2 March 2018)

Functions, squences

Aim: developing the ability for recognising relationship, feature description, decision-making about the pieces – if they have the given properties.

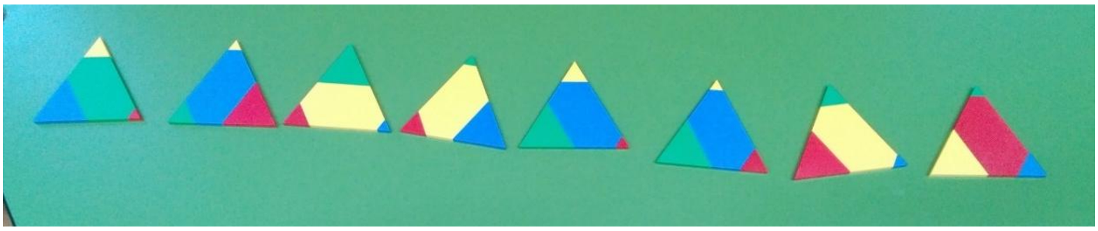
Selection based on given aspects (grades 1-4.)

The kids continued the sequence form the **triangle package**. Rule: the connected pieces in the sequence must be the same coloured and the sized triangles.



Sequences based on personal preferences (grades 1-4)

The kids created sequences choosing the rules in groups. E.g. in the sequence the colours of the triangle angles were repeated in periods: yellow, yellow, green, green.



Building plane shapes (getting geometry experience, grades 1-4.)

They created a mosaic based on the give aspect. Rule: the connecting points had to be identical in colour and size.

Our experience: the kids were working intensively

They learnt the following concepts: repetition, periodic, sequence

They got experience in comparison, observation, constructing and periodic repetition

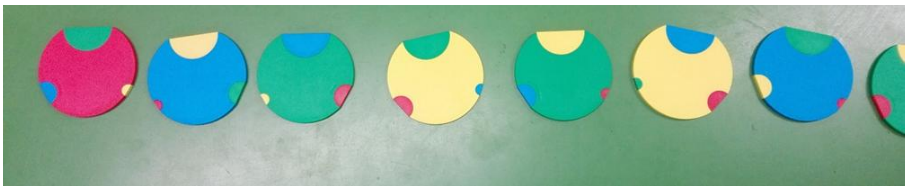
The sixth lesson (9. March 2018)

Functions, sequences

Aim: developing the ability of relationship recognition, feature description, decision-making about the pieces – if they have the given properties.

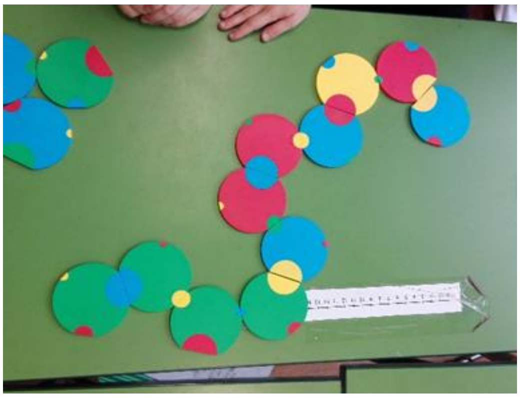
Creating sequences based on given aspects

The kids continued the sequence using the circle package. Rule: the connected pieces in the sequence must be repeated by the base colour – blue, yellow, blue, yellow



Building plane shapes (getting experience in geometry)

They created a mosaic based on the give aspect. Rule: the connection points had to be identical in colour and size.



Our experience: Creating the mosaics made our students work intensively again.

They got experience in: repetition, periodic sequence, arc, semicircle, curved line, angle, with a hole, without a hole

They got experience in comparison, observation, constructing and periodic repetition

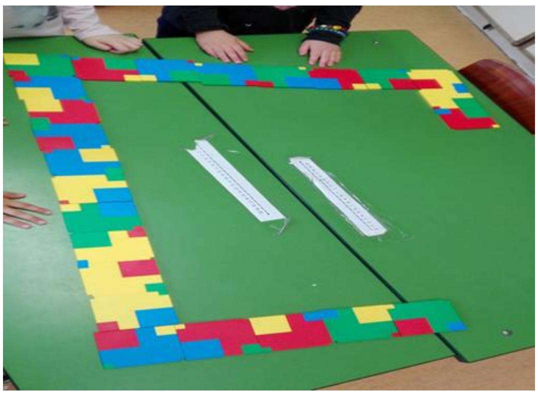
The seventh lesson (23 March 2018)

Functions, sequences

Aim: developing the ability for recognising relationship, feature description, decision-making about the pieces – if they have the given properties, compliance practising

Sequences created based in given aspects (grades 1-4.)

We were using the elements of the square package as dominos. The kids were working in pairs. They chose a piece from a bag and put it to the sequence based on given aspects. The rule: the next piece in the sequence must be identical in colour and size. Every single pair was working with 12 square elements.



Our experience: We had to revise the domino rules before we started to solve the task. The kids did not know in which direction they had to place the pieces. The rules of the previous mosaic task made the students confused.

They learnt the following concepts: domino, placing-joining the elements

They got experience in comparison, observation and compliance

The eighth lesson (6 April 2018)

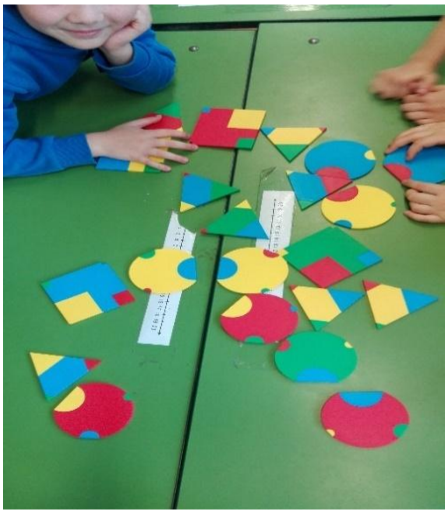
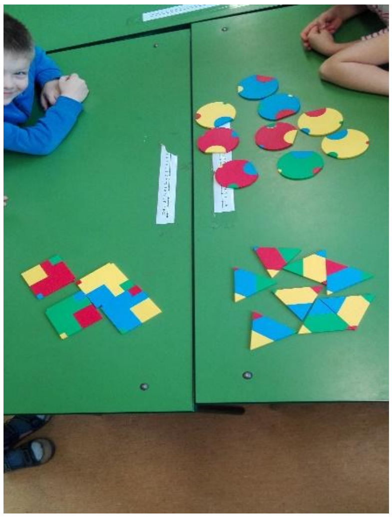
Functions, sequences

Aim: developing the ability of relationship recognition, feature description, decision-making about the pieces – if they have the given properties.

The kids first revised continuing sequences from every set (square, triangle, circle). When they were done in a rotation system they continued another sequence at the next table.

The kids created mosaics based on given aspects using different packages.

They selected the pieces:

Our experience: In addition to skill development, solving the tasks was also highly effective in developing collaborative ability and mastering organizational tasks.

The students revised all the concepts as they were working with each package.

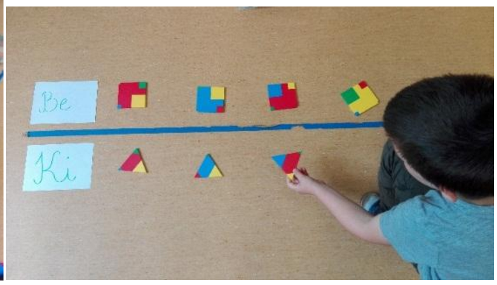
The ninth lesson (13 April 2018)

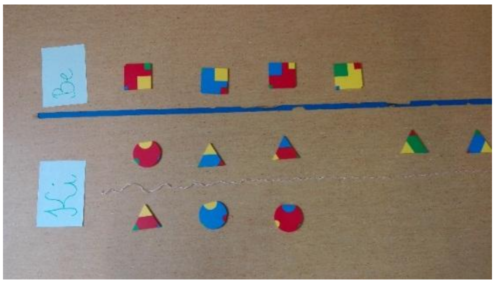
Functions, sequences – game with rules

Aim: developing the ability for recognising relationship, feature description, decision-making about the pieces – if they have the given properties, memory developing

Using a machine model for creating rules

We played first frontally, then in groups a game called “a machine which can create rules”. The kids’ job was to figure out what the machine would do with the pieces which were previously put into it. We performed the process with a mechanical machine model to help their imagination. Next, we inserted the pieces, which were related into a chart. **Rule**: The machine makes a triangle from the square which was put into it keeping its original colours. Next the machine converts the triangle into a circle, but it still does not change the colours.





Our experience: The kids matched the pieces which were related very quickly after we discussed the rules. For defining them they used their geometrical knowledge for e. g.: square, triangle, circle, they also created a new word: "unanglising" – meaning removing angles and making it round

They learnt the following concepts: convert, identical, as big as, difference, similarities

They got experience in comparison, observation and in recognising the related elements

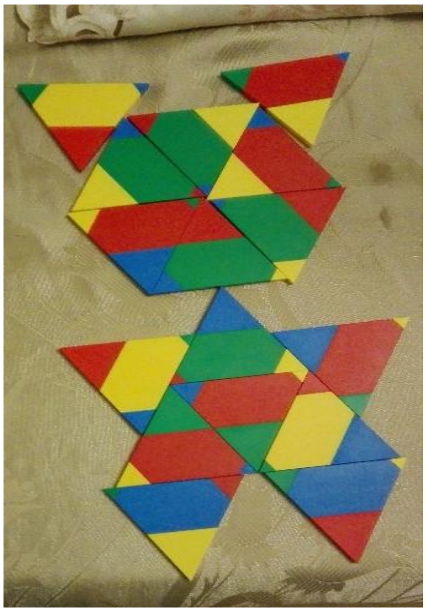
The tenth lesson (20 April 2018)

Getting experience in Geometry – constructing plane shapes

Aim: shape recognition, creativity, observation ability, comparison ability and aesthetic sense development

Creating mosaic based on a pattern (a photograph or a picture) – grades 1-4.

We enriched the kids' geometric knowledge and experience. Shapes (a bird, a teddy bear, a fox and a ship) created by the kids at the 4th lesson were projected onto the whiteboard while the kids had to remake them using the packages.

Our experience: They enjoyed the tasks very much, but they got tired at making the 3rd shape as all the shapes were varieties of bright colours and complicated patterns. They had to observe many features of one basic element to solve the tasks without mistakes.

They learnt the following concepts: identical, the same size, differences, similarities, rotation, flat, plane

They got experience in comparison and observation.

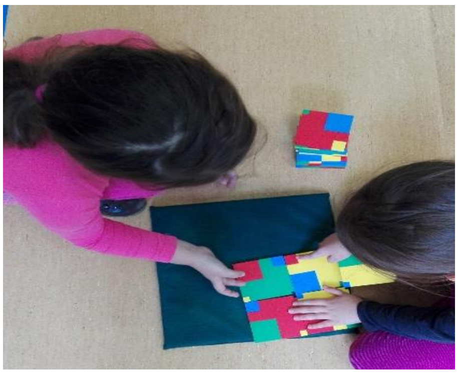
The eleventh lesson (27 April 2018)

Getting experience in Geometry – constructing plane shapes

Aim: to experience the way how the elements can be placed by sides and angles

Flooring (grades 1-4.)

The kids used the packages as floor boards. They covered different plane surfaces without leaving any gaps. We used the square package as the first. Working in pairs the kids covered seat cushions, in groups of four they covered tabletops. Together the whole class covered a huge area in front of the whiteboard.



Our experience: They enjoyed the task very much. At this task they did not have to pay attention to the small details of the basic elements. They only inserted the element side by side joining the edges.

They learnt the following concepts: insert, side, edge, cover, fit, plane

They got experience in covering plane areas.

The thirteenth lesson (4 May 2018)

Getting experience in Geometry – constructing plane shapes, geometric quantities and measurements

Aim: getting experience in how the elements can be inserted side by side joining their angles and sides

Flooring (grades 1-4.)

The kids used the pieces of the **triangle** package as floor boards. In pairs they covered seats cushion. In groups of four they covered tabletops. Together the whole class covered a huge area in front of the whiteboard.

Our experience: They enjoyed the task just as much as the one at the previous lesson in the reason that they did not have to pay attention to the small details of the basic elements. They only inserted the element side by side joining the edges.

They learnt the following concepts: insert, side, edge, cover, fit, plane

The thirteenth lesson (11 May 2018)

Getting geometry experience – constructing plane shapes, geometric quantities and measurements

Aim: getting experience in how the element can be inserted side by side joining their angles and sides, explaining area by some activities

Flooring (grades 1-4)

The kids used the elements of the circle package as floor boards. First, they tried to cover a seat cushion. They assumed that with curved surfaces it is impossible to cover a plane area without leaving gaps.

Covering surfaces: estimation, measuring (grades 1-2)

In group work the kids measured the surface of the cushions with square, circle and triangle elements. **Rule**: The basic elements could not lie on a bigger surface than the cushion. They had to cover the cushion as much as possible.
In addition to getting experience in Geometry they also got experience in comparison. They compared from which elements they would need the most and from which the least. First, they estimated the size then they measured. After the quick solution the groups could choose what they wanted to measure.

They enjoyed free measuring the most when they could measure what wanted.

They got experience in covering plane surfaces and in measurement.

The fourteenth lesson (18 May 2018)

Functions, logics, getting experience in Geometry

Aim: Practising what the students have learnt at the previous lessons

The students could choose from the tasks. The function of each task was to revise a previous topic. We broke the class into 6 groups. The groups had different tasks. The kids went to another desk to get another task as they finished. They solved the following concept: " a machine which can create rules", they created mosaic from the triangle package freely (based on the rule which they made themselves), they created tangrams based on given topics, they remade shapes based on a picture, they measured the area.

They learnt the following concepts: insert, side, edge, fit, estimate, measure, plane surface

They got experience in cooperate in groups

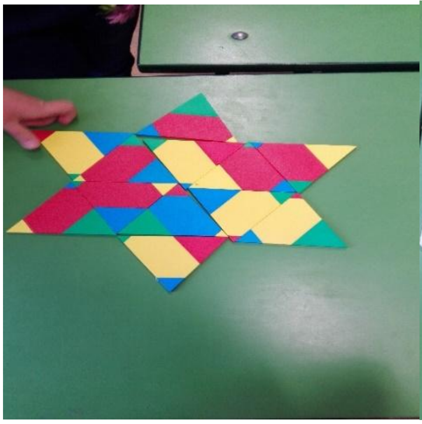
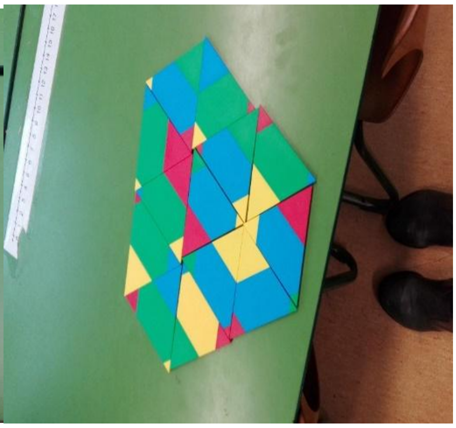
The fifteenth lesson (25 May 2018)

Getting experience in Geometry – constructing plane figures, diagram logic – selection, arrangement

Aim: developing creativity while they were working freely, improvement of the aesthetic feel, recognition of relations

Creating freely

The kids choose the packages (triangle, square and circle) which they wanted to work with. They could also combine the packages and create shapes totally freely.

Stocktaking

The kids arranged the elements, put everything on their place. The arrangement required recognition of the logical connection between the elements.

Our experience: the kids found it easy to arrange the elements. They recognised the logical connections between them very quickly.

They got experience in selecting, arranging and making difference between the basic elements in shapes and colours.

III. Possibilities how to use the game packages in grades 1-4

1. DIAGRAM LOGIC

1.1 classification (devision)
/creating rules – feature description, decision-making about the pieces – if they have the given properties, negotiation, coding/
1.1.1 Selection based on personal preferences 1-2.
1.1.2 Selection based on given aspects 1-2.
1.1.3 Correcting the mistakes in the selection 1-2.
1.1.4 Continued a selection 1-2.
1.1.5 Stocktaking
1.2 Logics / formation of thoughts via statements and decisions /
1.2.1 Deciding about the truthfulnes of the statements 1-4.

2 FUNCTIONS, SEQUENCES / developing the ability of recognising relationship /
2.1 Sequences based on personal preferences 1-4.
2.2 Sequences based on given aspects 1-4.
2.3 Using a machine model for creating rules 1-4.

3 GEOMETRIC EXPERIENCE

3.1 Constructing plane figures 1-4.

3.1.1 Building mosaic freely 1-4.

3.1.2 Building mosaics based on a picture – copying 1-4.

3.1.3 Building mosaic based on given aspects 1-4.

3.1.4 Tangram – building shapes based on given aspects (an animal, an object, symbol, designing a flag etc.) 1-4.

3.1.5 Creating patterns in sequences – recognition of periodic repetition 1-4.

3.1.6 Playing domino in pairs 1-2

3.1.7 Flooring – covering a plane surface without leaving any gaps – getting experience in fitting sides and angles 1-4

3.1.8 Building plane figures

Building triangles, rectangles, polygons from the given elements 1-3.

Building plane figures with arc – concaves 2-3.

Building shapes based on reflection building other ones not based on reflection 2-4. Building a plane figure with the least number of sides a plane figure can have from six elements 1-3.

3.2 Transformations

3.2.1 Drawing reflection, colouring it 1-4.

3.2.2 Modelling rotation 3-4.

3.2.3 Building a rotated shape from triangles 3-4.

3.2.4 Modelling offset 3-4.

3.3 Geometric quantities and measurements

3.3.1 Estimating and measuring the area of the surfaces 1-2.

3.2.2 Calculating the area of each shape on the square element, the side lengths are given by the teacher 4.

3.3.3 Calculating the area of each colour on the plane figures built from the square package

4. COMBINATORY /Improves seeing the details of things/

4.1 Mosaic flower

4.2 In how many ways can we place a triangle, a square? 2-4.

4.3 In how many ways can we put next to each other 2, 3 etc. triangles, squares? 2-4.

4.4 In how many ways can we join two elements which are identical shapes? 2-4.

5. ARITHMETIC, ALGEBRA

5.1 Reading fractions, making fractions

IV. Our experience in using the game packages with 1-4th graders

The fact that the 1-4th grader kids enjoy working with the packages and they get motivated when they use it was clear for us immediately when we used the kits for the first time. The shapes and the colours of the elements were nothing new for them as they used another logical kit right in the first month of the school year. The shapes of that logical kit and PUSE packages are identical: triangle, circle and square. The PUSE packages provide more ways of assembling than that logical kit as one more colour is printed on the PUSE base forms: they are yellow, blue, green and red.

The properties above make the game set suitable for selecting, classification, arranging and coding. These activities are important in making concepts. The kids become able to describe features, make decisions about the pieces: whether they have the given properties or not. Based on our experience these types of tasks are effective in the first and second grades, but the 3-4 grader kids can also come up with some new selecting ideas which the teacher may not have thought about before.

In the first grade we develop many skills by collecting and drawing line patterns, e.g. fine motor skills, observation, the ability to recognise the relationship between things. The packages were used in the first grade for the activities above, and later they became a tool for sequences and rule games. We were able to apply this way of use with older students as well, especially in the second grade, where it was a very impressive demonstration tool.

The use of the game family was the most effective in getting geometric experience. In the first grade, the children formed expressions from their own experience, such as curvature, fit, and connection, which underlie the understanding and acquisition of the terminology easier which they will learn later. The activities with the kit provided practice in estimation, measurement of area and in covering plane surfaces with and without leaving gaps.

Dividing the side length of the basic element by two, we get a smaller and different colour shapes on it. Thus, our 3-4 grader students could easily assume the areas on the elements due to scale-shifting symmetry. They could calculate the areas of the individual elements not only by experience, but also by conclusions. The kit is ideal for modelling offsets and rotations in the same grades. The appropriate size of the elements makes it easy for children to observe the properties of geometric transformations.

For creating combinatorial tasks shape, size or combination of them might be the base which makes the task exciting for the children.

In the fourth grade, the game packages are very useful for illustrating fractions because of the symmetric characteristic of the sets. These tasks can be based on the colour of the square and triangle elements.

The repetition of the attention-concentration test, which is intended to examine the developmental impact of the game, was done by the children in May, after the fifteenth lesson. The results were compared with the January results. Both the experimental and control groups performed better. This was reflected in making less errors and the faster solution of the tasks. The reduction of errors was observed in both teams. The experimental group, on the other hand, produced better time results, which means they produced better results in less time. However, from this skill test, it is clearly not possible to assume that the team using the game developed better because there were no significant differences between the experimental and control group performance.

To sum up, the Polyuniverse game family sets can be suited to the topics of 1-4th grade mathematics very well. Due to its visuality and playfulness, it is perfect for developing basic skills and forming concepts.

V. THE PROJECT: GRADES 5-8

In addition to the experimental class (8) in grades 5-8, we tested the game in all the grades as well as in the talent management extracurricular classes, camps, individual tutoring classes and talent development ones.

We organized the sessions in a block form in the experimental class. In Hungary, there are schools where epoch education takes place and teaching is organised around projects. In our research it is therefore worth examining how to use the tool not in a weekly or monthly manner, but rather in a shorter but continuous use grouped around a certain theme. In grades 5-8 we tried using the game this way.

The experimental class is particularly weak in mathematics, difficult to motivate, but open to all novelties and experiments.

The concept

With the exception of the experimental class, we went the same way in every group: with minimum teacher instruction we 'let' the groups use the stock to see what the fantasy of kids would make of it. We then provided a systematic package of questions raised by the children in the experimental class.

The questions thus gathered cannot be grouped around a mathematical area as they are diverse and varied. However, we can provide a list of the areas these questions looked at:

1. Geometry, plane motions
2. Geometry, area, perimeter
3. Graph theory
4. Combinatorics
5. Logic, pigeon-hole principle
6. Constructions
7. Puzzles (e.g. tangram)

In all cases we did group work.

Experiences with group work: In this respect, we get “ready-made”, “pre-cultivated” groups of learners from this aspect. Since they do regularly groupwork in a lot of subjects they are used to it by grade 5. Both the organization of the lesson and the social situations are routine. There is therefore no significant development here.

The more noticeable development in this field was in the case of the experimental group bearing in mind that social cooperation develops because of group work because of a tool. There are tools and types of tasks that are suitable for group work organization. With these a child's social-cooperation skills might be improved. The tool tested is excellent for organizing group work.

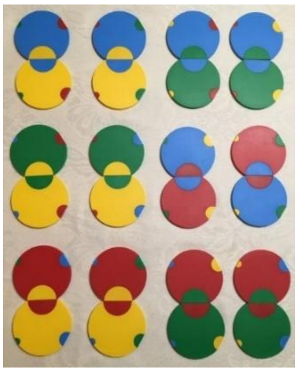
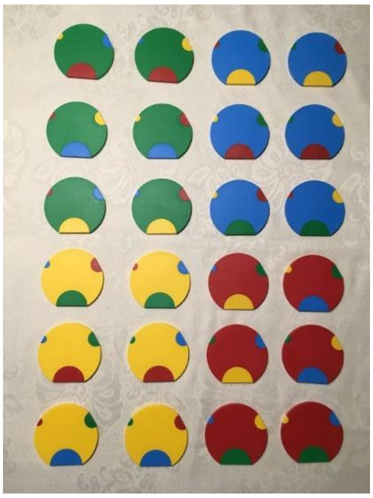
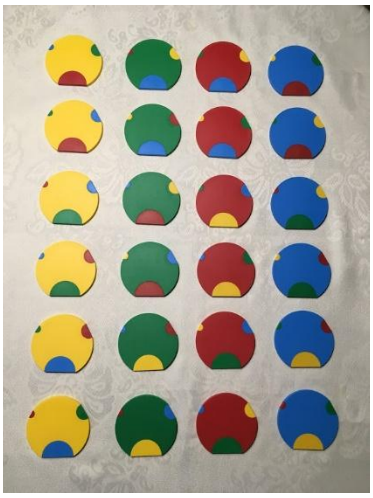
Classes in the non-experimental groups

As a first task I gave out incomplete sets in each group. The only information I gave away about the complete stocks were the following:

• there are no two identical elements,
• there are 4 colours on each item,
• these colours are blue, green, yellow, red,
• all possible items are included in the kit.

Then they had to make order and find the unnecessary or missing items.

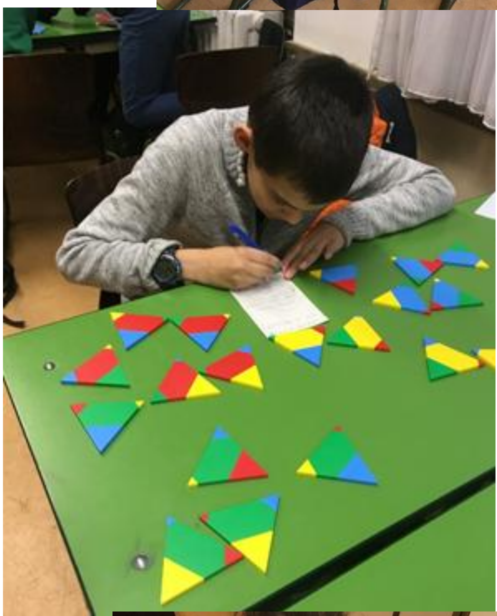
Experience: they formed order in a diverse structure.

They enjoyed working with the inventory with pleasure as they got acquainted with it.

They pose diverse questions ranging from very easy to very difficult. Their topics also exhausted the most unexpected areas.

After getting to know the set each group made puzzles for the other groups - once again without the instruction of the teacher, they were free to ask any question even if they did not know the answer. The questions were recorded on paper. Subsequently, they exchanged tables in a rotating system and sought answers to each other's questions. At the end of the sessions we discussed the most exciting questions.



Experience: Children's imagination is very diverse. They raised interesting and varied questions and eagerly awaited their peers´ opinions and answers.

Later on I took selected questions into some groups, and in other groups I followed a similar path as before. In the talent care camps, the children also played with the kit as a leisure activity. This is how a combinatorial and strategic board game was created (among other things) with hours of developmental work. (This is described in the task collection.)

The smaller age group (grades 4-5, 9-12 year-olds) were mainly interested in construction issues. Deployment of a part of the set or a complete set based on various conditions. There were kids who dealt with such a task for a few days (got the set for home use), and then sent the solution by mail.

Older students got excited about combinatorial and logical questions. In grades 7-8 (12-15 year-olds) the board game was the hit. Each game lasted for more than half an hour while thoroughly discussing potential situations (combinatorics!).

Classes in the experimental group

Classes took place in blocks, February (3x2 hours), March (2x2 hours) and April (2x2 hours). In the first block there was freestyle familiarizing with the game, then making and solving puzzles. In the second block we dealt with the tasks of other groups. In the third block we tested the board game.

Experience:

Most of the eighth-graders were happy to deal with the set and the puzzles, but here I met a student who did not get actively involved claiming it to be geometry, which is not his cup of tea. The others tried to convince him that it had nothing to do with geometry, but he was still wary of the work of others. He didn't get involved. This phenomenon shows that a tool is not enough to break the misconceptions that arise in a teenager.

The group was less interested in stockpile and construction tasks less and more in combinatorics, the pigeon-hole principle and other questions. The board game was the most popular of all.

VI. THE PROJECT: THE SECONDARY GRAMMAR SCHOOL

In the high school the game was tested in the 10th grade of science class. The science class is one of our four grammar school specializations. Here, students have more biology, mathematics, physics and chemistry classes, and the expectations in all these subjects are slightly higher than in other classes. In the ninth and tenth classes, all 4 science subjects are compulsory, starting from eleventh grade these classes become optional, so they can further specialise according to their field of interest. Most of the time, the class divides into two parts in their senior year, one half of them becoming doctors, a chemists or biologists, and the other engineers, physicists or mathematicians. More and more foreign universities are chosen for further education, so almost every year some are admitted to Oxford, Cambridge and other renowned European universities. It may be regularly observed that at the Fazekas school the students studying natural sciences study the most and that their subject average is the highest in the grade, meaning that they have a lot of hours, they are very diligent, persistent and tired.

Concept

When using of the game, I did not endeavor to fit it into the lesson. Unfortunately, the characteristics of the curriculum or my limited fantasy did not allow me to do so. So, apart from rare exceptions, the game was played on a particular day of the week and we dealt with mixed topics. I planned to take tasks from a larger topic each week and to get to know and deepen our already-given knowledge through the game in most cases meticulously guiding what you can do with the kit and how.

Topics:

1. Geometry
2. Graph Theory
3. Combinatorics
4. Probability Calculation
5. Game Theory

I did one class a week in the morning. The duration of the classes was 90 minutes involving 30 students. The five topics were not evenly distributed, geometry taking the lead probably also thanks to my distorted vision of the material in this direction.

In the following I will describe my part not lesson by lesson but broken into topics. The lesson by lesson organisation can be seen on the employer record sheet.

GEOMETRY

We went through the changes in area ratio and aspect ratios quickly, some words about volume ratios in general, as well as the length, area, volume, and other dimensional proportions. Then I gave them some easier and more difficult geometrical problems from the task collection. I wanted to focus on translating similarity into algebraic form, and how a task can have different solutions that differ in nature or how a problem can be further thought and the tips checked, or perhaps proven. We couldn't finish the topic. In my opinion, about 60 lessons of geometry could be put together based on the set. In absence of this only the initial steps were taken.

Experience:

Some elements of the kit are well-suited to the similarity issue, to center and peripheral angles, to trigonometric triangle computation and to other trigonometric calculations. But such lessons may be conducted only if the students know the stock. It is not a good idea to start with this, as the kit has not been designed for this purpose. This is an extreme use of the set. It can help if the stock items are built with GeoGebra beforehand.

GRAPH THEORY

Since we add graph theory equation to pieces of the game, e.g. each piece should be a vertex and two vertices shall be joined together. If they have exactly the same two parts (colour and size), then we get a vertex-transitive graph that can be well suited to the Hamiltonian Paths or Hamiltonian Circuits. This can be enough for starting to think of it or to imagine the idea of its existence. Starting from this basic concept, I began to give lessons, but soon I had to realize that the students were not getting my intended goal. However, it was available for them to use the tools but unfortunately the number of the lessons started to be more than could be acceptable. Even though that entire topic has not been covered, I can imagine giving 20-30 lessons in this topic. Of course, the kit is also suitable for much simpler ones, which I unfortunately had previously done with other sets. For example, it can be perfect for the degree sum.

Our experience

The students enjoyed this part much better as they could work manually. However, the colours and the shapes of the elements may cause limitations in some topics. Thus, it can be used successfully only if using it with some other tools. For older students, I recommend getting started with the topic above or combinatorics to get to know the game.

COMBINATORICS

Of course, there were a lot of combinatorial problems, and I gave the students different tasks after a few simple, commonly discussed questions. For example, by using six triangles, assembly a hexagon so that in the middle there would be a solid, large hexagon. How many ways is it possible?

Our experience

Combinatory is what this game set is really good for. I really recommend starting with this topic to the teachers. There can be created plenty of interesting tasks in it and it doesn’t matter with which one you finish; it won’t make you feel that you haven’t reached any goal. As many questions you can set to the students that many issues they can suggest to deal with in addition. Another thing what I would mention is that you will not discover anything new, but it is perfect for mastering permutations.

PROBABILITY THEORY

Also, combinatorial problems have emerged from probability problems, but geometric probability problems were also mentioned. Examples of the above: What is the probability of a randomly selected element area from a set that the half of it at least will be green? If you choose 2 elements from a set and you put one on the other so that they are in one shape, what is the probability that the colours in the connection points will not be identical? We were dealing with such and similar problems for 2 hours.

Experience:

Although these two hours flew by easily and without problems, I still think that much more could have been done. The use of the sets in this case was not as important as in the geometry section. Thus, it was not actually necessary to take them in their hands. For this reason, I recommend working in this topic when students already know the elements of the sets very well and they know their features by heart. This is how they can solve these mathematic problems effectively.

GAME THEORY

The final big project I was working on were circle strategy games. The basis of these games were games created by Erika Jakucs's students which can be found in the collection of tasks.

Our experience

In this case using the set itself got high importance. The students were using it continuously. They rotated the pieces and put them on the right place. There is no easy-to-find winning strategy, but the question itself arises whether there is any strategy which leads you to win or does any strategy exist which leads you to not lose the game. We could talk about the differences, explore other games like this, talk about programming as a mathematical tool, or search for algorithm. Perhaps the only disadvantage is that it can be played with the circle package only, or if I saw it well, maybe even with the triangle.

Poly-Universe in School Education (PUSE) Erasmus+ Project

REPORT by Experience Workshop
FINLAND
by Nóra Somlyódy and Kristóf Fenyvesi

Experience Workshop ay
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CONTENTS:

1 Introduction

Experience Workshop (www.experienceworkshop.org) serves as the Finnish coordinator of Polyuniverse in School Education (PUSE) project.

During Phase 1 (01.10.2017. - 31.12.2017.) we were preparing our cooperation with Finnish schools. We identified the possible partners, contacted school principals and teachers and established the framework of our cooperation. Two of our partner-schools in Jyväskylä (Finland) have decided to collaborate in our project. They involved four class-teachers and three classes in the required age groups, and three more classes which serve as control groups in the project.

As part of Phase 1, our project member, who works in the project as teacher, Dr. Kristóf Fenyvesi took part in the Transnational Project Meeting in Hungary in December 2017.

In Phase 2 (01.01.2018. - 31.05.2018.) we realized the PUSE-surveys with the participating classes, control groups and teachers in the beginning, in the middle and in the end of the period. Three PUSE classes and three control groups took part in the pupil surveys, and in the middle of the project our teacher-coordinator also filled in the questionnaire prepared for the PUSE project. In Phase 2 the Poly-Universe tool and related methodology was introduced to the participating teachers and pupils, new PUSE math-problems were developed and tested in PUSE classes. Feedback from pupils and teachers was collected.

PUSE Schools in Finland

- **Jyväskylä Christian School:** Jyväskylä Christian School is located in Jyväskylä. It is owned by a private NGO foundation. There are 430 students in the school. The school's community respect Christian values, but they are multicultural, embracing several ethnic and religious backgrounds. The school has strong regional, national and international networks. This is an international school with about 110 immigrant students from 25 different countries with 19 different languages.

  Teachers participated in PUSE project: *Jukka Sinnemäki,* elementary school teacher, who has been nominated among the best 25 teachers of the world at the Global Teacher Prize 2018 (https://www.globalteacherprize.org/). *Juha Kyyrä*, elementary school teacher, who also serves as an educational coordinator in his school.

  Classes participated in PUSE project: 3rd grade + control group 4th grade; 6th grade + control group 5th grade.

- **Viitaniemi School:** Viitaniemi School is in the city of Jyväskylä offering education from 7th to 9th grade. There are 440 students in the school. The school has English classes for students with international background, as well as autism education and preparatory classes for immigrant children. The school offers English language at A1, A2 level, Spanish and Russian languages, as well as B2 level in German and Spanish. **Teachers participated in PUSE project:** *Merja Sinnemäki*, mathematics, physics, chemistry and English teacher, a leader of the English program. *Leena Kuorikoski*, art teacher.

  Classes participated in PUSE project: 7th graders + another 7th grader control group.

2 Experience Workshops' Work Process in the PUSE Project

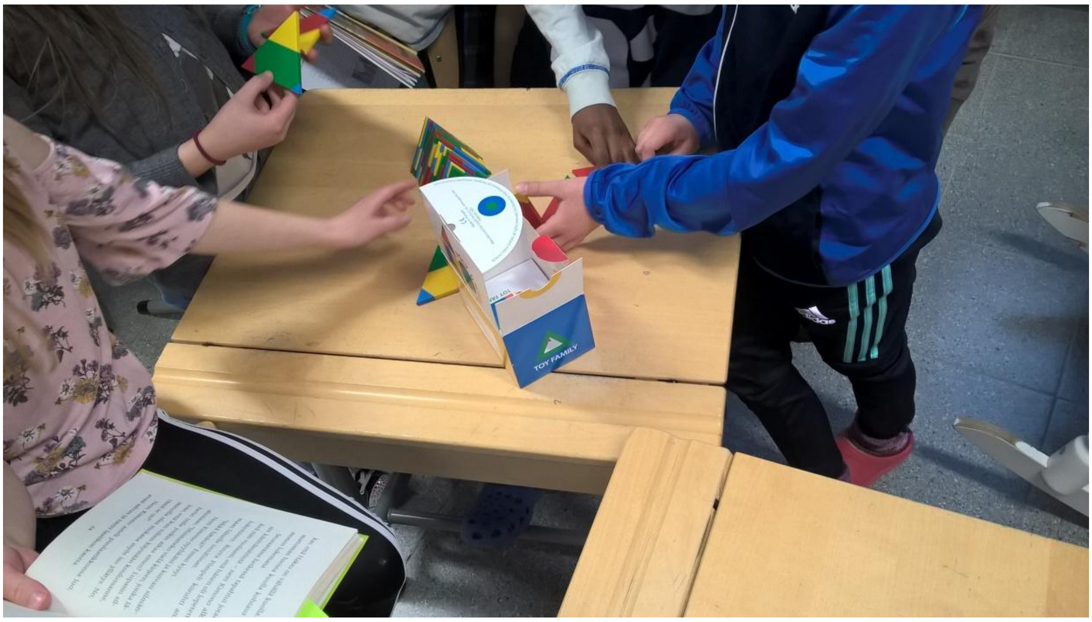
Phase 1 (01.10.2017 - 31.12.2017)
During October and November 2017 Experience Workshop took up regular contact with the coordinator Poly-Universe Ltd and all partner institutions, using the shared google drive database, Skype, phone and the google groups e-mail address.

We had a broad previous experience with the Poly-Universe tool and its use in mathematics education, but we had to reshape our knowledge according to the new PUSE project goals and also adapt it to the Finnish National Core Curriculum, to local learning and teaching conventions and to the school organizational environment. During October we were preparing the adaptation based on the Poly-Universe documentation. First, we gathered information about possible participating schools in Jyväskylä and selected Viitaniemen Koulu (Viitaniemi School) and Jyväskylän Kristillinen Koulu (Jyväskylä's Christian School) as participants in the PUSE project. We created a local implementation plan. Then we took up contact with both schools, which were more than ready to embrace the project. We organized the introductory meetings with school principals and interested teachers.

When principals and teachers got acquainted with the project, the local implementation plan was reviewed and refined with the help of local teachers. Then further discussions took place on the implementation and an agreement on project details was found. Based on the principals' suggestions, 4 volunteering teachers were selected from among the teaching staff; 2 teachers from each school. Having familiarized with the expectations of the schools as well, we prepared the Poly-Universe presentation for teachers.

On 03-04.12.2017 the first Transnational Project Meeting took place in Budapest, Hungary. Important administrative issues were clarified and project members could exchange their first ideas and plans with local schools about the implementation of the project. There were also special mathematics class visits to Fazekas-school, Poly-Universe demonstrations and an art & science exhibit in the Saxon Art Gallery, which served as the meeting place.

Mid-December we proceeded to introduce the Poly-Universe tool to teachers and also trained them on how to use it in the classroom. Pupils got acquainted with the tool on the following day.



Phase 2 (01.01.2018. - 31.05.2017.)

In January we gathered information about the local curriculum (reviewed pupils' mathematics exercise books) and compiled a comprehensive Poly-Universe collection of PUSE tasks (until May). We worked on how to integrate Poly-Universe exercises into local curriculum and introduced Poly-Universe in the classes of the 3 age groups.

The testing of the Poly-Universe tool was carried out between January and June in both Finnish partner schools in 3 different age groups. Each group worked with those Poly-Universe problems, which were prepared and introduced under the coordination of Experience Workshop's expert and teacher and the volunteering local teachers. There were 2-4 Poly-Universe meetings for 1-2 hours each in all age groups during this period - the frequency of meetings depending on local curricular possibilities and also following the group dynamics of children. Pupils in all age groups were encouraged to participate in the co-creation of the problems.

The PUSE measurements were conducted in close cooperation with local PUSE teachers. They collected the research permits and organized the survey procedure in the 3 classes + 3 control groups. The measurements in all 3 age groups were taken in two stages.

We collected the following materials during the period:

- Measurements materials, evaluation
- Observations of teachers and pupils
- Methodological description and suggestions
- New mathematical PUSE tasks
- Photos and video archives etc.

Finally the Poly-Universe study was written and edited.

On 31.05-01.06.2018 Experience Workshop organized the second Transnational Project Meeting in Jyväskylä, Finland. The main goal of the meeting was to prepare the Poly-Universe educational methodology. During the meeting, the assignments related to the next steps of the project were distributed, the implementation of the project was monitored, the achieved results were presented by the partners.

Both Finnish PUSE schools were ready to host our group to hold the meeting. This possibility also brought the group close to the pupils: PUSE project partners could directly experience how open and creative Finnish pupils were in using the Poly-Universe tool. On 31.05.2018 in the Viitaniemi school local PUSE teachers, Leena Kuorikoski and Merja Sinnemäki greeted us and later PUSE pupils held us presentations. We used the day on the one hand for project assessment and reflection, where each partner presented the methodological materials and experiences gained during the 2nd project phase in 20 minutes. Secondly, each partner presented their PUSE study and the assessment documentation.

On the second day of the meeting, 01.06, which took place in Jyväskylä's Christian School, Poly-Universe experience workshops were held for 1st-6th grader pupils. Our PUSE-teachers from this school, Jukka Sinnemäki and Juha Kyyrä joined us on that day. Teachers participating in the meeting offered pupils various Poly-Universe fun programs (they could see approximately 60 pupils in work). On that day the discussion of the Poly-Universe methodology and workbook took place. We discussed the possible formats of the workbook, the many possible ways in which it could be used in and outside of the classroom. We agreed on the sample exercise sheet which working units will use in order to compile the individual Poly-Universe exercises. Deadlines were discussed for the creation of the workbook.

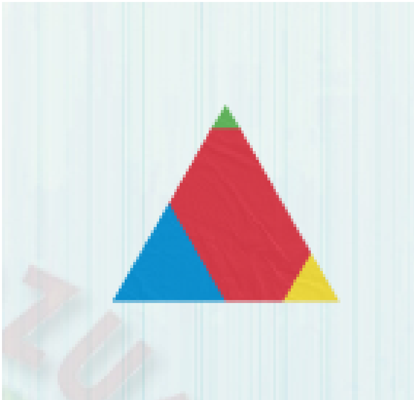
3 Collection of PUSE Math-problems

The problems collected below were developed in conversation with PUSE expert, Dr. Eleonóra Stettner.

Fractions 1

We only use the triangle shapes from the Polyuniverse set. How they are put together does not matter, only the sizes and forms count this time.

1 triangle:



Q (Question): How many different shapes can be put together out of two triangles?

A (Answer): Only one, a rhombus.

Q: What portion of this shape is a triangle?

A: 1/2

Q: How many different shapes can be put together out of three triangles?

A: Only one, an isosceles trapezium.

Q: What portion of this shape is one triangle?

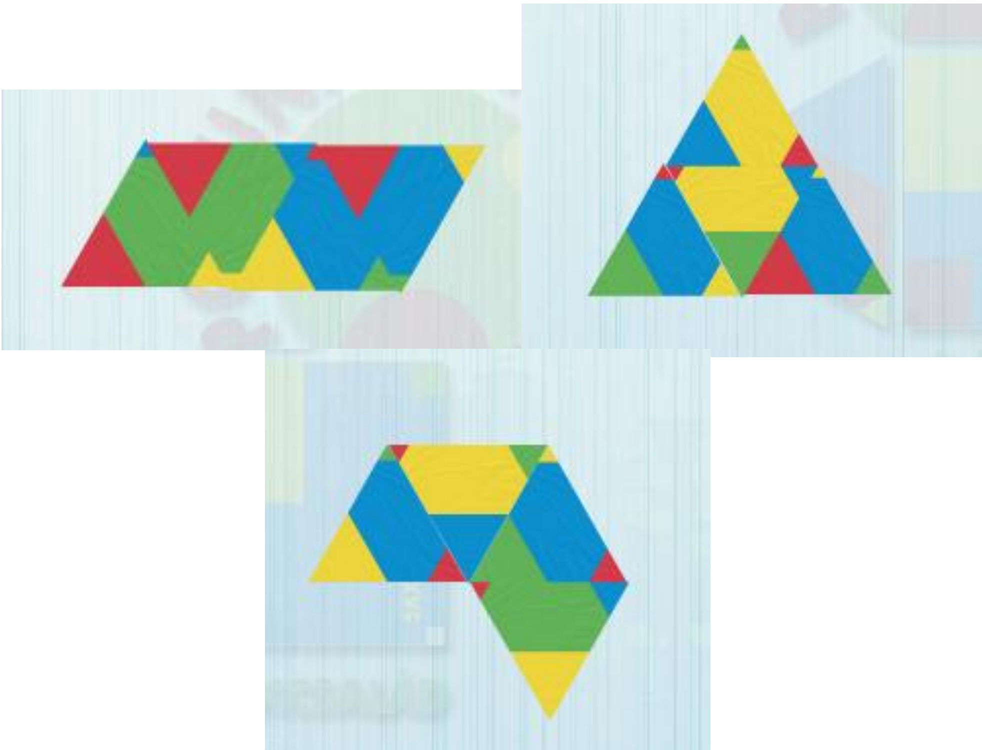
A: 1/3.

Q: What portion of this shape is the previous shape made up of two triangles?

A: 2/3.

Q: Out of four triangles, how many different shapes can be created?

A: The three below



Q: What portion of these shapes is one triangle?

A: 1/4

Q: What portion of this shape is the shape made up of two triangles?

A: 2/4 or 1/2.

Q: What portion of this shape is the shape made up of three triangles (c)?

A: 3/4

Q: What if we join 4 triangles?

A: In the case of four triangles the story gets more exciting. It's possible to create three significantly different shapes (one bigger triangle, a parallelogram, and even a concave hexagon might appear). Here, you might refer to concave and convex shapes, on the level of the age group: "Is the bunny able to hide from the fox?" In which case yes, in which no?

Q: What proportion is the triangle of the shape made up of four triangles (quarter), the rhombus (half), and the trapezium consisting of three triangles?

Fractions 2

The problems can be solved with any shapes in the Polyuniverse tool kits.

Put all the elements of the kit upon each other, let's call this a 24-storey building.

Then build from these elements as many 2-storey houses as possible, using up all the pieces.

Q: How many 2-storey houses is it possible to build?

Q: What portion is the height of a 2-storey house of the 24-storey house?

The same with 3, 4, 6, 8, 12-storey buildings.

24 is a good number, since it has a lot of divisors.

Q: How could you build identical 5- and 7-storey houses by using up all the elements? (Here, the question of divisibility comes in indirectly.)

Alternatively, putting the elements on top of each other, you might combine all the pieces of the Polyuniverse triangle set into one pattern. Then divide this into 3, 4, 6, 8, 12-piece segments, each segment consisting of two elements.

The above questions can be posed here, too.

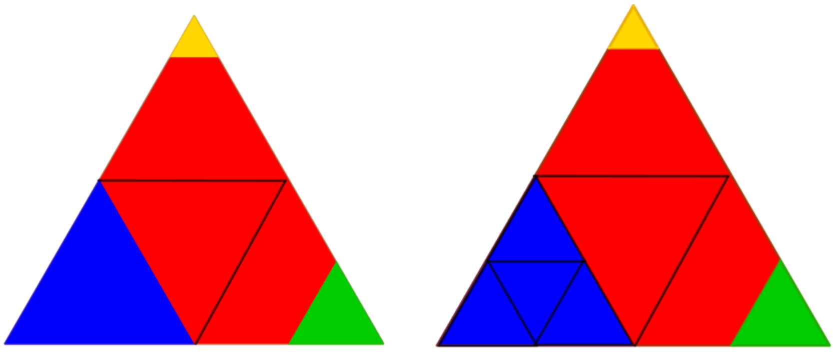
Fractions 3

Q: First let's make estimations. Who makes a better guess? What do you think, which one is bigger, the sum of the three small triangles' (blue, green, yellow) area or the red domain's area?

A: Students will probably guess "red".

Q: Let's try to estimate, how many times the red domain is bigger.

Q: Estimate, what percentage is the sum of the three small triangles' (blue, green, yellow) area of the entire triangle's area? And what about the red domain?



Q: What proportion of the entire triangle is the area of the blue triangle?

A: 1/4. Well visible in the image above.

Q: What percentage is the area of the blue triangle of the entire triangle?

A: 1/4=0,25=25%

Q: What proportion of the blue triangle is the green triangle's area?

A: 1/4. Well visible in the image above.

Q: What percentage is the area of the green triangle of the blue triangle?

A: 1/4=0,25=25%.

Q: What proportion of the entire triangle's area is the green triangle's area?

A: 1/16. You can divide the left side image's blue triangle (congruent with the 4 blue triangles) into 4 triangles congruent with the green triangles.

Q: What percentage of the entire triangle's area is the area of the green triangle?

A: 1/16=0,0625=6,25%.

Q: What proportion of the green triangle’s area is the yellow triangle’s area?

A: 1/4.

Q: What percentage of the green triangle’s area is the area of the yellow triangle? A: 1/4=0,25=25%.

Q: What proportion of the blue triangle’s area is the yellow triangle’s area?

A: 1/16.

Q: What percentage of the blue triangle’s area is the area of the yellow triangle?

A: 1/16=0,0625=6,25%.

Q: What proportion of the entire triangle’s area is the yellow triangle’s area?

A: 1/64.

Q: What percentage of the entire triangle’s area is the area of the yellow triangle?

A: 1/64=0,015625=1,5625%.

Q: What proportion of the entire triangle’s area is the blue, green and yellow triangle’s area in total?

A:

$$\frac{1}{4}+\frac{1}{16}+\frac{1}{64}=\frac{16}{64}+\frac{4}{64}+\frac{1}{64}=\frac{21}{64}$$

Q: What percentage of the entire triangle’s area is the blue, green and yellow triangle’s area in total?

A: 25%+6,25%+1,5625%=32,8125%≈32,8%

Q: What proportion of the entire triangle’s area is the red triangle’s area?

A:

$$1-\frac{21}{64}=\frac{64}{64}-\frac{21}{64}=\frac{43}{64}$$

Q: What percentage of the entire triangle’s area is the red triangle’s area?

A: 100%-32,8%=67,2%

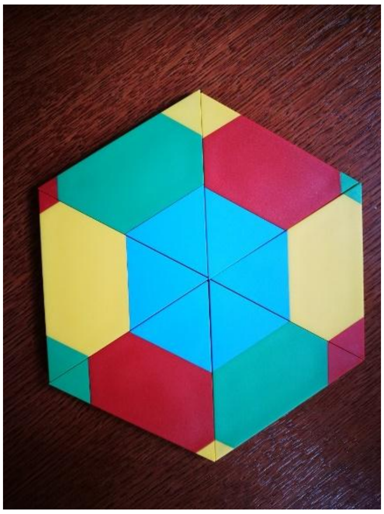
This means that the red area is a little bit bigger than twice the sum of the three little triangles' area.

Q: Let's prove that in the hexagon constructed according to the image below, the sum of the blue, yellow, red and green area is the same, that is, each of the colours occupy a quarter of the hexagon's area.

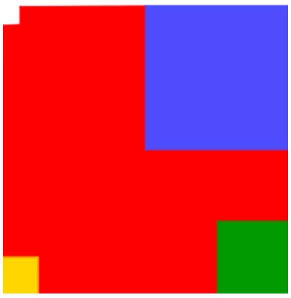
A: There is no need to calculate, since this is a thought-provoking exercise, it might however come as a good wink that the blue triangle makes up a quarter of the entire triangle.

Q: What percentage of the entire figure is occupied by the colours each?

A: 25%-25%-25%-25%



Segment, angle, perimeter, area



Let's measure the sides of the square.

Q: How many of the sides have to be measured in order to calculate its perimeter?

A: There might be some arguments because of the tiny missing square. Give the length of the side and the perimeter both in cm and mm. Let's measure the sides of the small squares (blue, green, yellow) and calculate their perimeters. How many sides does the red domain have? We want to measure the perimeter. Do we need further measurements?

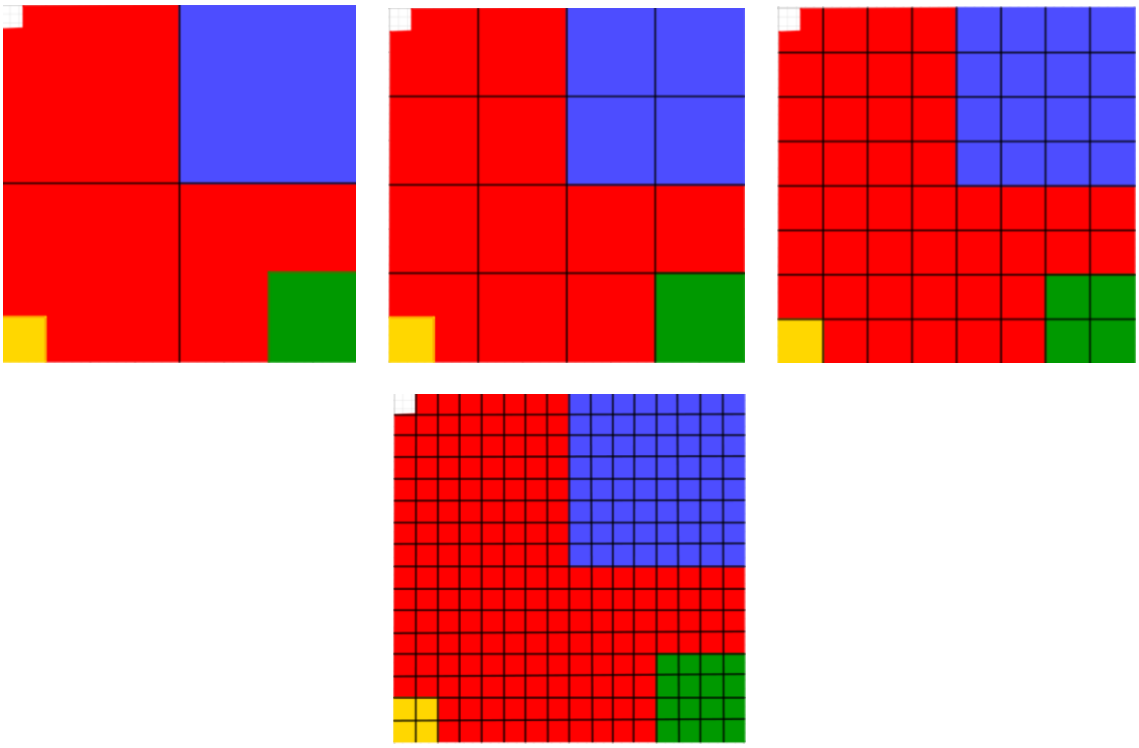
Q: Are there acute angles, right angles, obtuse angles in the square shape?

A: There are only right angles.

Q: Show me where. How many are there in total?

Q: If the tiny white square is disregarded– how many blue squares do you need to make up the entire shape?

Q: Let's choose for the sake of the game instead of the $1m^2$ square the blue square and let's call it "1 blue". We can conclude that this way the entire shape is "4 blue", "16 green", "64 yellow", "256 missing" square.



Q: How many squares should be put next to each other in order to create a segment longer than 1 dm out of the sum of the square sides?

A: 2.

Q: How long is the so created segment?

A: 18 cm.

Q: How many squares should be put next to each other in order to create a segment longer than 1 m out of the sum of the square sides?

A: 12.

Q: How long is the so created segment?

A: 108 cm.

Q: Let's measure one side of the triangle. How many of the sides have to be measured in order to calculate the perimeter? Give the length and the perimeter both in cm and mm.

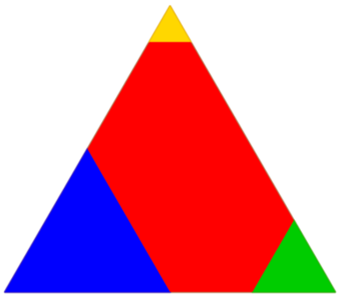
Q: Measure the sides of the small triangles (blue, green, yellow) and calculate their perimeter. How many sides does the red shape have? We want to calculate its perimeter. Do we need further measurements?

Q: Are there acute angles, right angles, obtuse angles in the triangle shape?

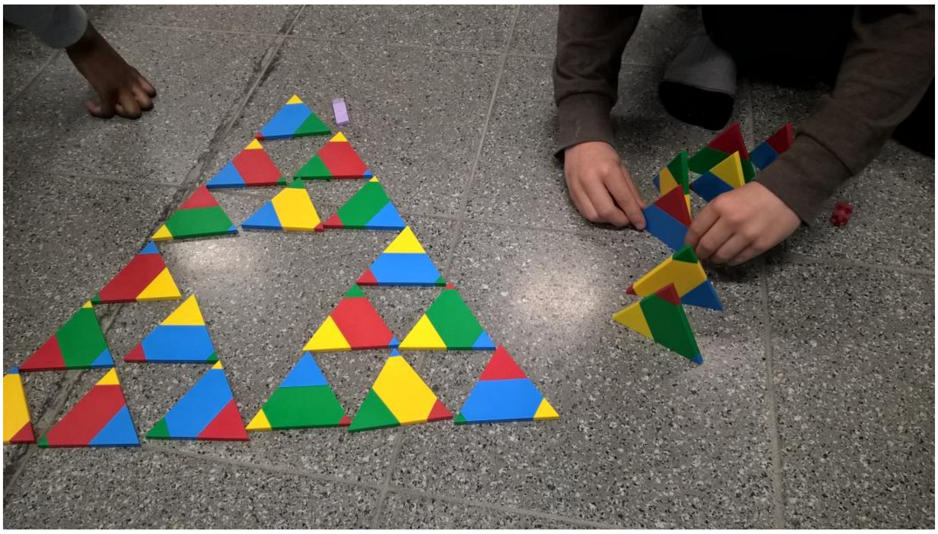
A: There are only acute and obtuse angles, no right angles.

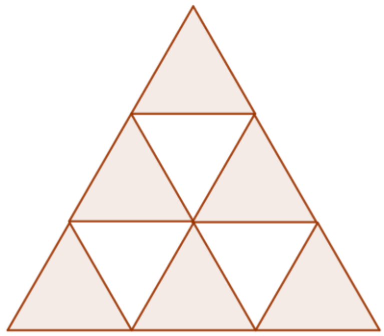
Q: Show me where. How many are there in total?

Q: How many blue (green, yellow) triangles do you need to make up the entire shape?



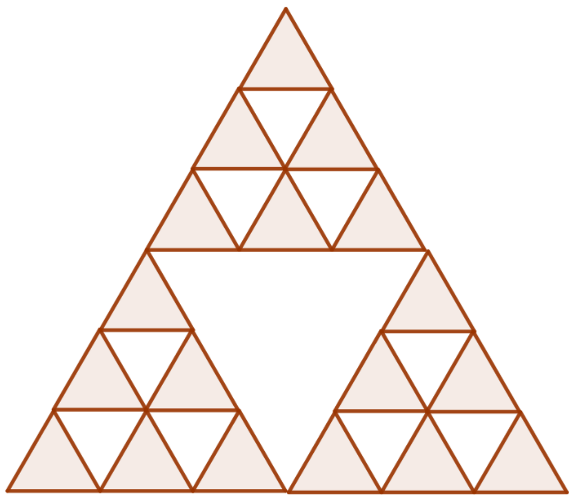
The Area of a Pattern





Q: Put together the above pattern out of Polyuniverse triangle shapes. How many triangles does it consist of?

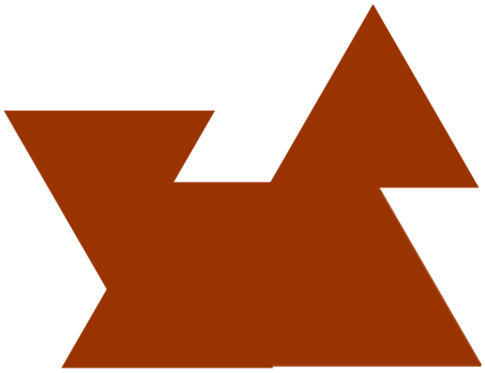
Q: Continue the exercise with a longer row. How many rows are you able to put together using one box of triangles? How many triangles will this pattern consist of?



Q: We have put the above drawing three times next to each other. How many triangles does it consist of? Consider mentioning that this is a famous shape – Sierpinski Triangle, Sierpinski Gasket or Sierpinski Sieve –, named after Waclaw Sierpinski.

Q: How is it possible to continue, how many boxes would be needed to complete the next part? Etc.

Q: How many triangles is this "puppy" made up of? Construct it.



Q: Invent and put together further interesting figures and count the number of triangles they consist of.

What is in a Box? Think Inside the Box!

Main color	**Big-sized semicircle**	**Middle-sized semicircle**	**Small-sized semicircle**
yellow	green	red	blue
		blue	red
	blue	green	red
		red	green
	red	green	blue
		blue	green
green	yellow	blue	red
		red	blue
	red	blue	yellow
		yellow	blue
	blue	red	yellow
		yellow	red
blue	red	green	yellow
		yellow	green
	yellow	red	green
		green	red
	green	red	yellow
		yellow	red
red	blue	yellow	green
		green	yellow
	yellow	blue	green
		green	blue
	green	red	yellow
		yellow	red

Enclosing Arcs, Chords

Q: How many chords and arcs enclose a Polyuniverse “circle”?

A: 3, 3.

Q: What if two Polyuniverse circles are joined together along 1-1 chord of the appropriate length?

A: 4, 6.

Q: How many “Polyuniverse circles” should be joined in order to have on the border: 6 chords?

A: 4 in chain or 6 in closed circles.

Q: How many “Polyuniverse circles” should be joined in order to have on the border: 15 arcs

A: 5 in chain.

Q: How many “Polyuniverse circles” should be joined in order to have on the border: 13 arcs?

A: Not possible.

Q: Look for the rule.

Number of "circles" put into a row	Number of enclosing chords	Number of enclosing arcs
1	3	3
2	4	6
3	5	9
4	6	12
5	7	15
6	8	18
n	n + 2	3n
closed circle 6	6	18

Angles for Triangle, Square, Sum of the Polygon's Angles

We use Polyuniverse triangle and square shapes. Only the shapes matter, not the way they are put together. We should disregard the tiny missing square on the square shape, too.

Q: What is the measure of the triangle's angles?

A: 60°.

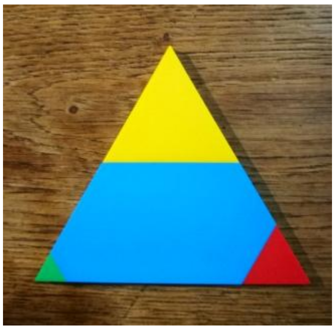
Q: What is the sum of its angles?

A: 180°.

Q: What type of triangle is this?

A: Equilateral.

Q: What ican we say about its sides?



A: Equal.

Q: Can you combine the triangles into a square? Even in different ways?

Q: What is the measure of the angles of a quadrilateral?

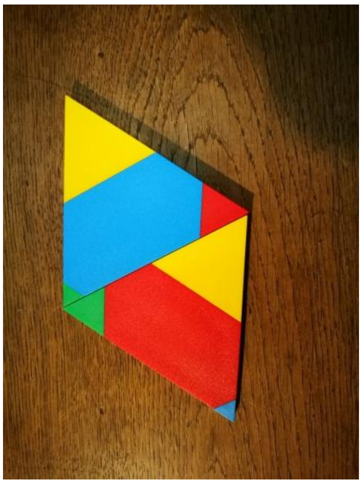
A: 60°, 120°.

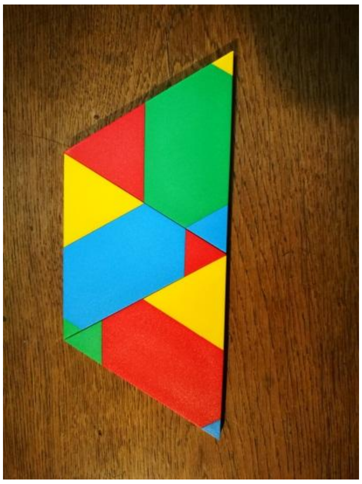
Q: Why?

A: Because either they are angles of a triangle or its double.

Q: What is the sum of the angles?

A: 360°.



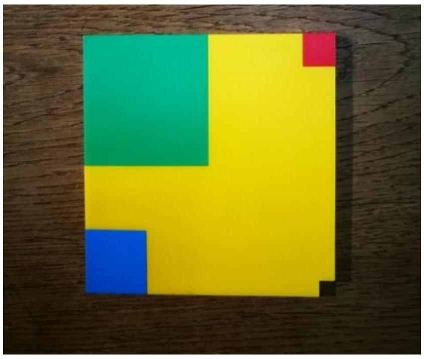


Q: What is the measure of the angles of a square?

A: All the four are 90°.

Q: And what is the sum of its angles?

A: 360°.



Q: Can you combine the square and the triangle into a pentagon? What is the measure of its angles?

A: 60°, 90°, 60°+90°=150°.

Q: And what is the sum of its angles?

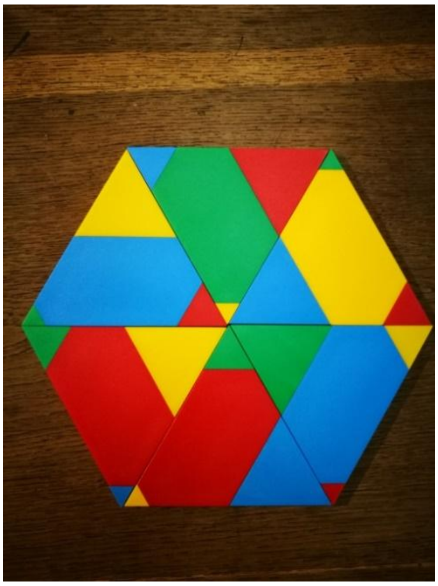
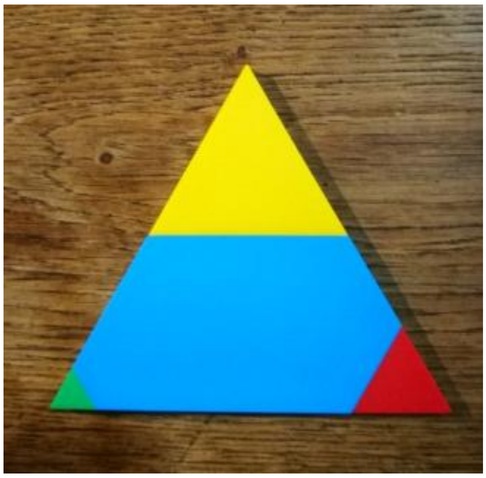
A: 540°.

Q: Combine triangles into a hexagon. Compare it with the unicolour hexagon created within the triangle. In what aspects are they similar?

A: All of their angles are 120°.

Q: How do they differ?

A: One of them has equal sides, the other does not.

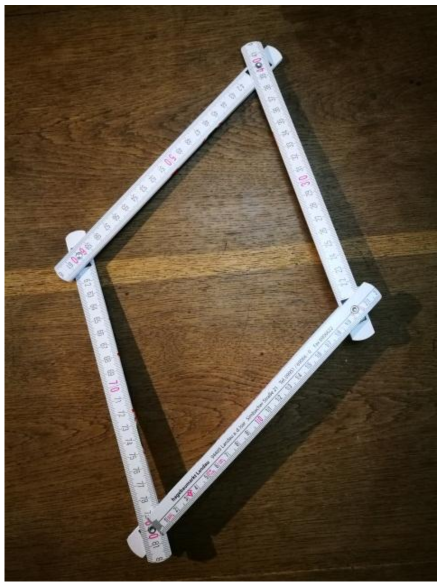



Q: Is it true that if the angles of a quadrilateral are equal, so will be the sides?

A: Counterexample: square, rectangle.

Q: Is the reverse statement true? If the sides of a quadrilateral are equal, so will be the angles?

The following illustration will help:

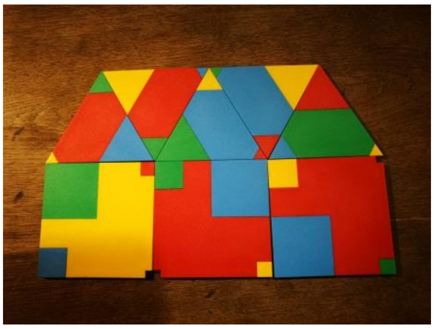
Q: What can we say about the above statement in the case of a triangle, a pentagon or a hexagon?

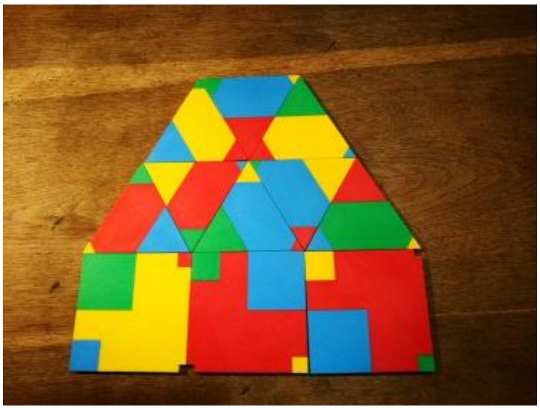
A: True in the case of a triangle: if the sides are equal, the angles are equal, too. And the other way round: if the angles are equal, so are the sides. This is an equilateral (regular) triangle. In the other cases none of the statements are true.

Q: Combine Polyuniverse triangles and squares into as diversely shaped “houses” as possible. Check in all of the cases how big the angles have become. What is the sum of the angles of each “house”? How many sided polygons are the “houses”?

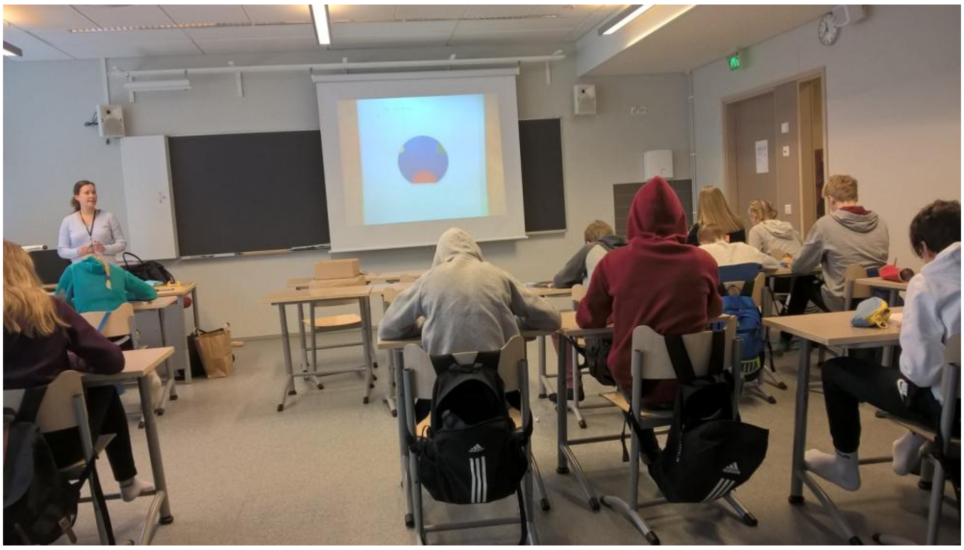
Some examples:

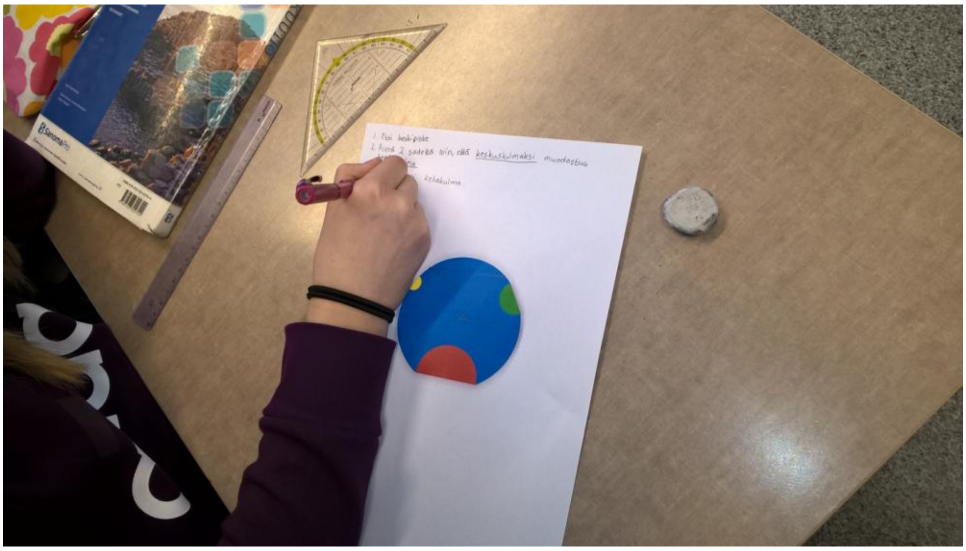







Circle





Q: The Poly-Universe set is made up of three basic shapes: triangle, (almost) square, and (almost) circle. The colors are: red, yellow, green, and blue. The shapes are colored in every possible way using these four colors. We use the sets containing the "circle" shaped forms. The diameter of the “circles” is 9 cm, the thickness is 0.5 cm, having the same color on both sides. There are three semicircles attached to the boundary of the circle in directions making 120^o angles with one another, the largest has radius that is half of the original circle, for the medium it is one fourth, and for the smallest it is one eighth. The diameters of the three semicircles cut off three segments from the original circle so the basic form is not exactly a circle. So this shape is bordered by 3 arcs and 3 line segments, so they can stand on their line segment parts in a stable way, as kids tried it.

All of the exercises should be calculated in 3 ways: parametrically, then with r=8 cm basic circle radius (this way it’s easy to halve and calculate three times successively), and in original size, r=4,5cm

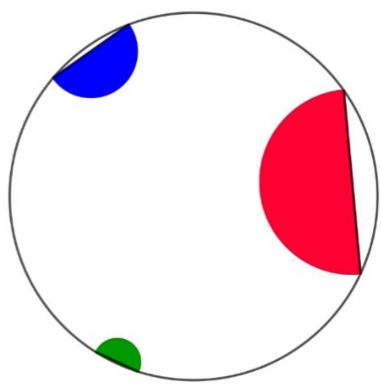
Q: Let’s calculate the area of circle of radius r, r=8 cm, r=4,5 cm.

A:

$$T = r^2\pi$$
$$T = 8^2\pi = 64\pi = 201{,}06\ cm^2$$
$$T = 4{,}5^2\pi = \frac{81}{4}\pi = 63{,}62\ cm^2$$

Q: Let’s calculate areas on the “almost” circle polyuniversum shape. Why is this shape not an exact circle, but an “almost” circle?

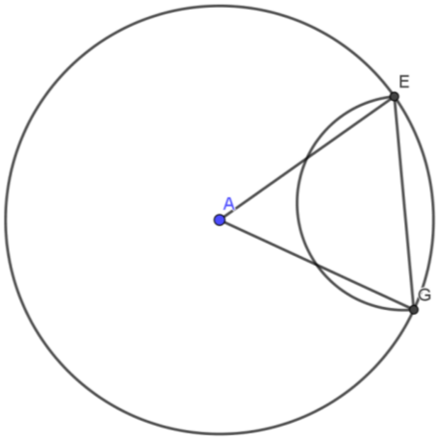
A: circle segment, semicircle, chords - enclosed by 3 arcs and 3 chords

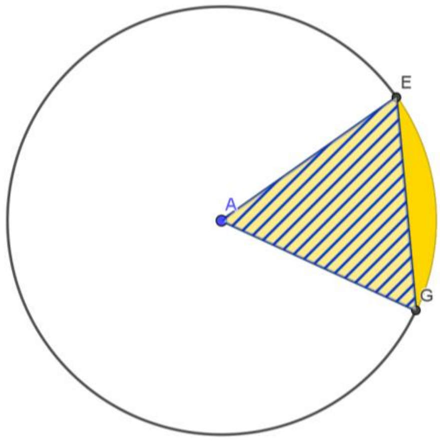


Q: Let’s calculate the area of the coloured semicircles on the figure above.

Q: Let’s calculate the area of the shapes enclosed by the perimeters of the coloured circles and the arcs of the big circle. What’s the name of these shapes?

A: Circle segment.





To make the shapes more transparent, the outline of the biggest semicircle is still visible in the figure on the left, while in the right figure not anymore.

Q: Let’s calculate the area of the yellow shape.

A:

Let’s deduct the area of AGE triangle from AGE circle sector.

AGE triangle is equilateral (regular), so the angle at A is 60°.

$$T_{AGEcircle\ \ sector} = \frac{r^2\pi \cdot 60°}{360°} = \frac{r^2\pi}{6}$$

$$T_{AGEcircle\ \ sector} = \frac{8^2\pi \cdot 60°}{360°} = \frac{8^2\pi}{6} = 33{,}51\ cm^2$$

$$T_{AGEcircle\ \ sector} = \frac{4{,}5^2\pi \cdot 60°}{360°} = \frac{4{,}5^2\pi}{6} = 10{,}60\ cm^2$$

We use the trigonometric area formula to calculate the area of the triangle: $T = \frac{a \cdot b \cdot \sin(\gamma)}{2}$.

Here the two sides are the radius of the original circle.

$$T_{AGEtriangle} = \frac{r^2 \sin(60°)}{2} = \frac{r^2\sqrt{3}}{4}$$

$$T_{AGEtriangle} = \frac{8^2 \sin(60°)}{2} = \frac{8^2\sqrt{3}}{4} = 27{,}71\ cm^2$$

$$T_{AGEtriangle} = \frac{4{,}5^2 \sin(60°)}{2} = \frac{4{,}5^2\sqrt{3}}{4} = 8{,}77\ cm^2$$

Now the calculation of the area of the circle segment is as follows:

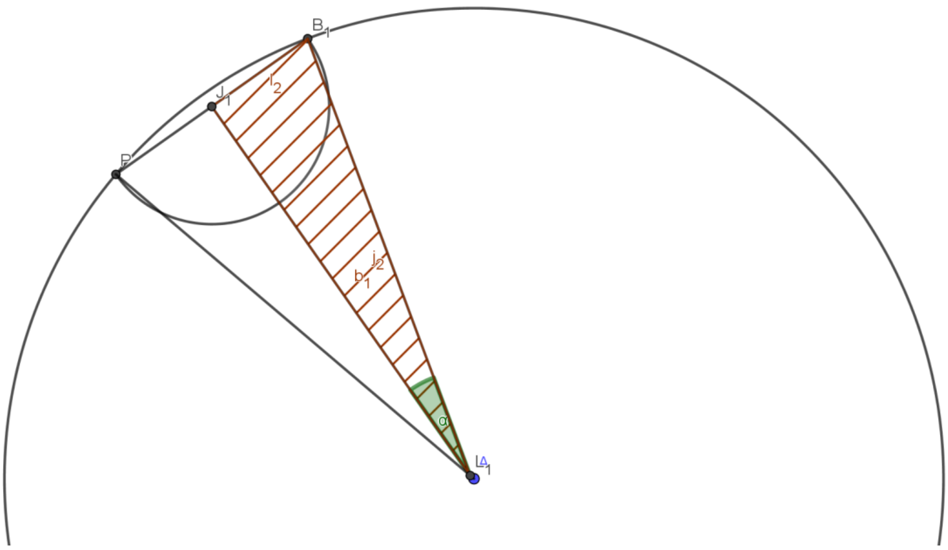
$$T_{circle\ segment} = T_{AGEcircle\ sector} - T_{AGEtriangle} = \frac{r^2\pi}{6} - \frac{r^2\sqrt{3}}{4}$$

$$T_{circle\ segment} = T_{AGEcircle\ sector} - T_{AGEtriangle} = 33{,}51\ cm^2 - 27{,}71\ cm^2 = 5{,}8\ cm^2$$

$$T_{circle\ segment} = T_{AGEcircle\ sector} - T_{AGEtriangle} = 10{,}60\ cm^2 - 8{,}77\ cm^2 = 1{,}83\ cm^2$$

Q: We can calculate the area of the two smaller circle segments in a similar way.

A: The second circle segment calculation. First, the central angle should be determined.



Calculate the green coloured angle based on the brown shaded right-angled triangle. Its double will be the central angle, with the help of which the area of both the circle section and the triangle can be determined.

$$\sin(\beta) = \frac{\frac{r}{4}}{r} = \frac{1}{4}$$

$$\beta = 14{,}48°$$

The central angle: $2\beta = 28{,}96°$, doesn't differ a lot from 30°, but 30° is a conceptual mistake!

Using the above formulas we calculate the area of the second circle segment, only the angle will differ.

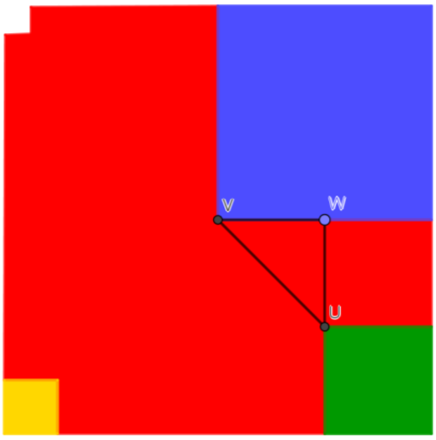
Q: Calculate the third similarly.

A: We calculate the “inner” area of the polyuniverse so that we deduce the areas of the three semicircles and the three circle segments from the area of the circle.
Twice that big central angles go together with twice that long arcs but not twice as long chords.

Q: How can we make out the centre of a polyuniversum circle?

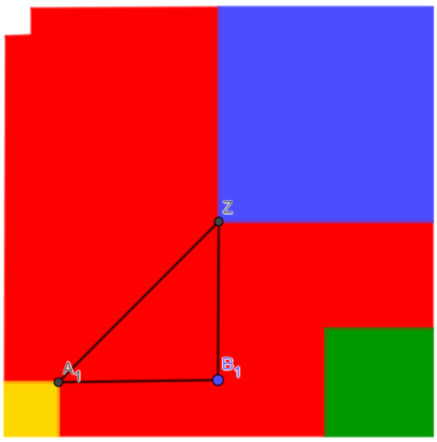
A: The longest chord is exactly as long as the radius, that's the reason why the longest chord and the radii drawn into its endpoints create a regular (equilateral) triangle. Framing the circle can go like this: I pin my compass into the endpoints of the longest chord and open as wide as the longest chord. Then I draw arcs from both endpoints, and the two arc's intersection will be the circle's centre. Then I connect the centre of the circle with the other arc's endpoints. Let's measure the angle created by these two radii.

Pythagorean theorem

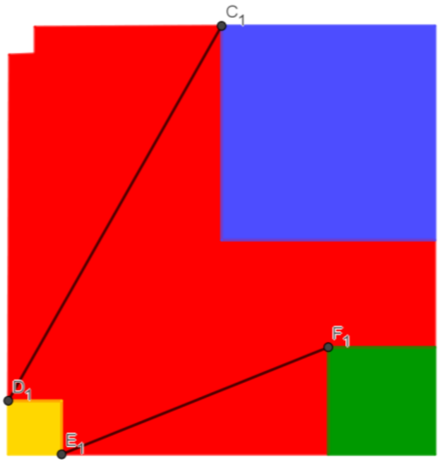


Q: Let's measure the length of UV segment.

A: First, we measure the side of the square. Then we draw UW and WV segments (which are the legs of the right-angled triangle), in order to create a right-angled triangle. The right angle is positioned at W point. UW makes up the quarter of the square's side, just like WV. Based on the Pythagorean theorem, $UV^2 = UW^2 + WV^2$, the UV segment's length can be calculated.



The length of A_1Z segment is measurable in a similar way. Now the right-angled triangle's leg is $a - \frac{a}{2} - \frac{a}{8} = \frac{8a-4a-a}{8} = \frac{3a}{8}$ part of the square's side. In case the square's side is 8cm, 8-4-1=3cm, $A_1Z^2 = 3^2+3^2$, $A_1Z^2 = 18$, $A_1Z = \sqrt{18} = 4{,}24$ is the value rounded to two decimal places. The segment connecting Z and the point of the „hole" not fitting onto the side can be measured similarly.



Not only the hypotenuse of isosceles right-angled triangles can be measured, but one can even regard the segments on the above graphic as hypotenuses of right-angled triangles. The legs can be measured easily if the length of the square's side is known.

Q: How many segments of different length can you draw in such a way that their endpoints coincide with 2-2 points of the four squares (which are altogether 16)? Look for as many as you can. The length can be measured using Pythagorean theorem. Are there equal segments among them, too? How many? Which are those?

A: The triangle and the circle can be used to exercise the Pythagorean theorem as well.

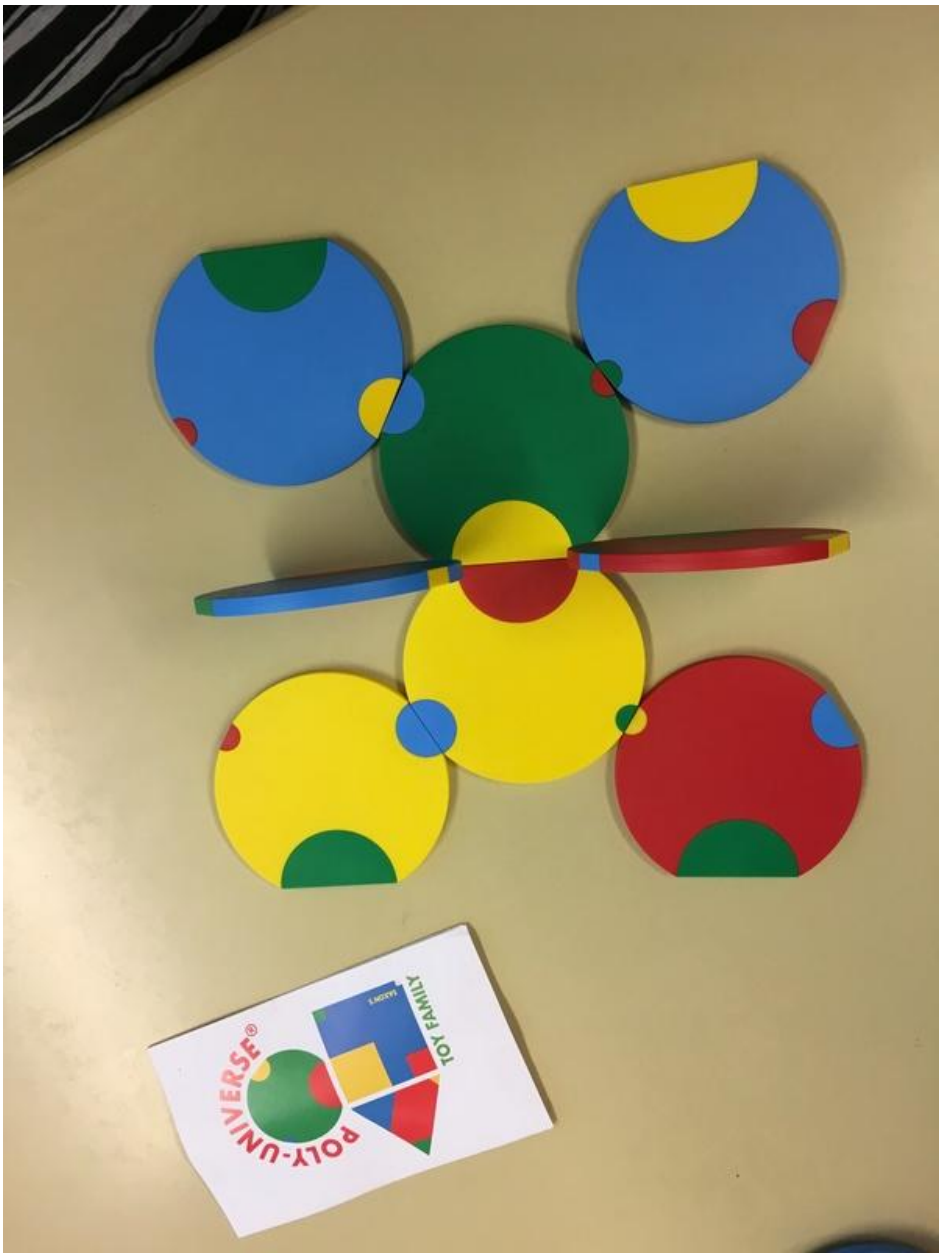
Probability

Considering probability calculations, experimentation makes sense in this age. E.g.:
Students work in pairs. Each pair gets a 24-element full Polyuniverse set. Mix the elements in a plastic bag, so that students don't see it, when they draw a piece. After each draw, students tell their partners the colour of the centrepiece. The partner takes notes. Each pair should draw at least 10 times. In the end, summarize the colours drawn by the pairs and then those of the whole classroom, too, at the blackboard.

Then complete the same experiment so that only the pieces with red and blue centre remain in the plastic bag.

Complete the same experiment another time, so that the pieces with red, blue and green centres are left in the bag.

Q: What do you observe? Approximately, what proportion of all the drawn pieces will be red centred in the first, second and third case? When is the chance for a red centre bigger? The more you experiment, the better it is.



PUSE

2nd BROCHURE
Nafarroako Ikastolen Elkartea - NIE
(Federación Navarra de Ikastolas)
2018/06/15

INDEX

1.- CONTEXT OF PUSE STUDY

1.1 The PUSE project is being implemented in two Ikastolas: Arangoiti and Zangoza.

Both of them are small schools

Both of them are in rural areas of Navarre

Zangoza Ikastola has 187 student

Arangoiti Ikastola has 77 student

1.2 The PUSE team in Nafarroa consists mainly of 5 members:

A general coordinator: Irene Lopez-Goñi

An expert in mathematics: Jesus M. Goñi

And three teachers: Idoia Legarreta (Zangoza ikastola: level C)

Garazi Larrañaga (Arangoiti ikastola levels A y B)

Guillermo Guerrero (support)

1.3 The PUSE project is being applied in three levels:

- A: students aged 6-8 years (A-PUSE)
- B: students of 10-12 years (B-PUSE)
- C: students of 14-15 years (C-PUSE)

- **Number of student in each age group and school:**

Ikastola	A	B	C
Arangoiti	23	19	
Zangoza			17

- **Number of sessions (of 2 hours each) that has been carried out at each level of age**

Ikastola	A	B	C
Arangoiti (*)	10	10	
Zangoza			10

2.- PREVIOUS WORK MEETING

As an initial part of the project, a series of meetings have been held with the different members of the educational community in order to present the project and indicate its objectives and interest. In each of the aforementioned ikastolas, the following was carried out:

- A meeting with the center's management team

- A meeting with the parents of the student who was going to participate in the project

- Two meetings with the teachers who were going to participate in the project

An amount of 10 hours has been used to prepare and carry on that part of the work

3.- DESING OF MATERIALS

3.1 In the meetings held with the teachers who were going to work directly with students in the project, the convenience of preparing a series of materials to facilitate the good development of the project was approved. To this end, templates were prepared for the tasks to be created for the students as well as for the help and evaluation of the same by the teaching staff

In Annexes I and II you can see these templates:

- Annex I (Template for the tasks of the students)
- Annex II (Template for the evaluation of the task by teachers)

3.2 With the intention of analyzing and organizing the tasks to be prepared, a spreadsheet was prepared that uses the following fields for the analysis of each task.

Age (A,B,C) A = 6-10; B= 10-14; C= 14-18
Time:
Shapes:

Mental process
Representation level:
Resources:
Contents of Maths:
Group organization:

3.3 Another template to create a Portfolio of students. (Annex III)

3.4 Digital versions of the PUSE pieces were created in order to use them in the creation of tasks. In Annex IV you can see a copy.

4.- DEVELOPMENT OF MATERIALS

4.1 Creation of the following materials:

(some examples of these materials are in the Annex V)

- A student book level A with 20 task
- A student book level B with 20 task
- A student book level C with 20 task
- A teacher book level A with 20 task
- A teacher book level A with 20 task
- A teacher book level A with 20 task
- A spreadsheet with the description and analysis of all the task .

5.- DURING THE EXPERIENCE

The experience that is related in this PUSE study has been developed between January and June and supposed the accomplishment of the following tasks.

5.1 Meetings of the PUSE team.

During the months of the experience, we held monthly meetings that were attended together with the professors who participated in the experience, the general coordinator and the expert in Mathematics.

The topics discussed in these meetings were aimed at seeing how the experience developed, sharing ideas, solving the small problems that arose and contributing new ideas to carry out the work in class.

At this moment we are collecting the ideas for improving materials and prepare the last Student's and Teacher's book.

5.2 During this period we have collect evidences of the work of students

Students at level A working with PUSE pieces

A student at level B working with his Task book

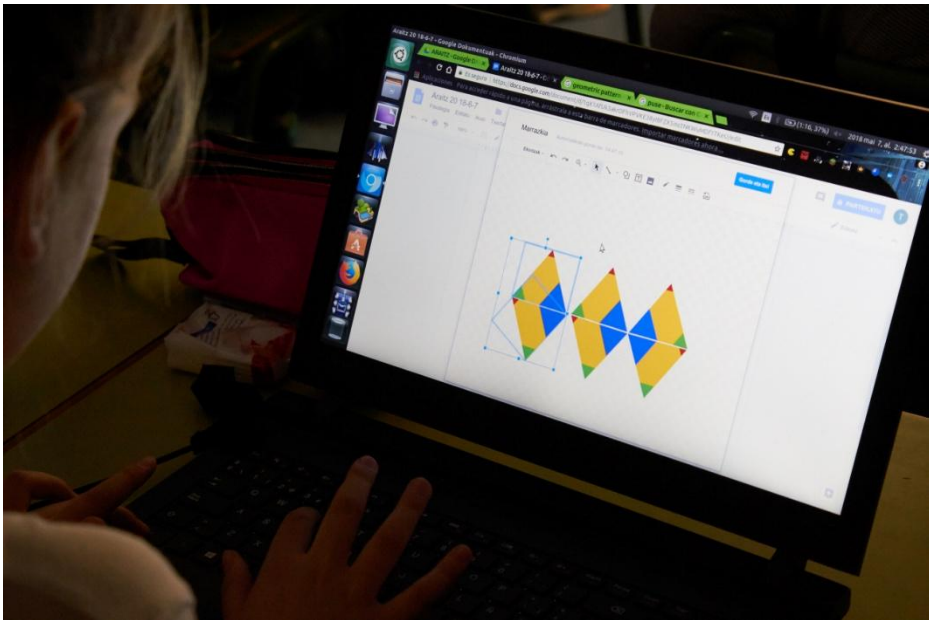
Students at level C working with their students books

5.3 PUSE and Google Docs and Draw.

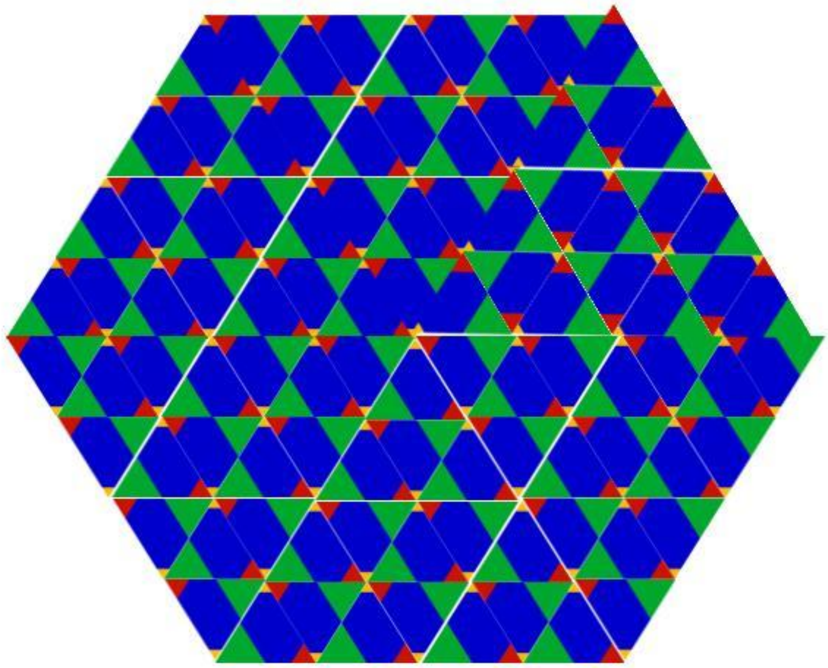
We have done a special work with the students at level B of the Arangoiti ikastola. In this experience students have worked with digital representations of PUSE pieces. The goal of the experience was teaching how to create a geometric design using PUSE pieces.

Each student had his or her own Google account and a special folder prepared with the digital task that they have to do.

One of this digital task can be seen in the Annex VI.



Here is a design created by a student using Google Draw



6.-COLLECTING STUDENTS' WORKS

At this moment (June 15) we are collecting the portfolios that are being prepared for students in order to analyze the work that they have done.

ANNEXES:

Annex I: Template for the task to be created.

<table>
<tr><td></td><td>Erasmus + PUSE Project
Ikastola (school):
Grade (A,B,C) /Age:
Task number:
Time:</td><td></td></tr>
<tr><td colspan="2">Name (student): …</td><td>Date: …</td></tr>
<tr><td colspan="3">Description:</td></tr>
<tr><td colspan="3">Remarks /Self evaluation:
………………………………………………………………………………………………………</td></tr>
</table>

Annex II: Template for for the evaluation of the task by teachers

	Erasmus + PUSE Project **Task number:** **Grade (A,B,C) /Age:** **Operation:** **Set:** **Language:** **Time:**	
Description:		

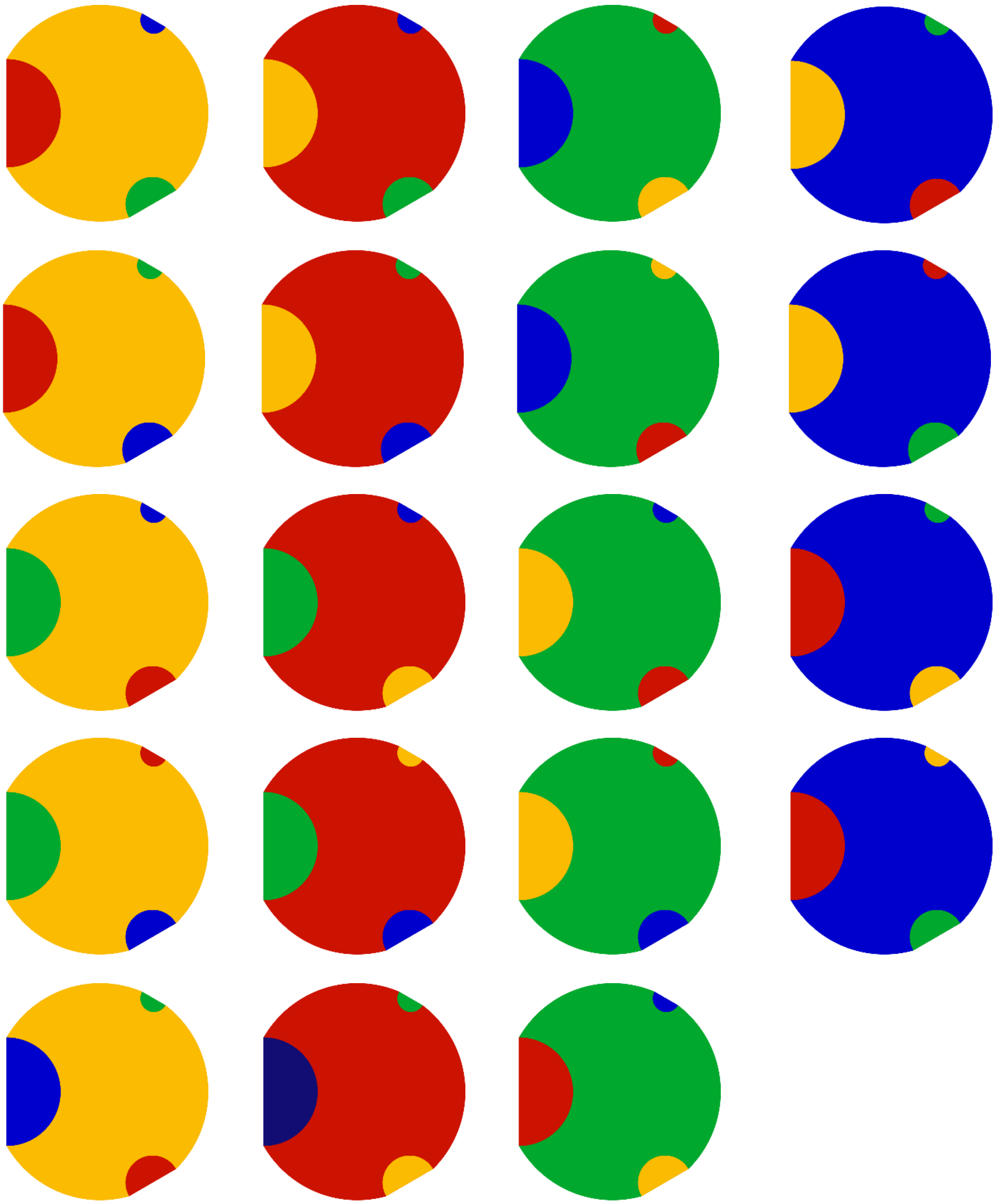
Evaluation of the Task (From 1, less positive, to 4, more positive)	**1**	**2**	**3**	**4**
Evaluation of the task/ Jardueraren ebaluazioa:				
Adaptation to age /Egokia adinerako:				
Time estimation/ Denboraren estimazioa				
Motivation grade / Motibazio maila:				
Learning value/ Balio didaktikoa:				
Percentage of success/ Arrakasta portzentajea:				
Adequate to the diversity of students/ Egokia ikasleen aniztasunarentzat:				
Favours creativity/ Sormena laguntzen du				
Comments/Oharrak: …				

Annex III:

	Erasmus + PUSE Project **PORTFOLIOA**	
SCHOOLYEAR: **SCHOOL:** **GRADE (A,B,C)/AGE:** **NAME:** **DATE:**		

Annex IV: Digital copy of PUSE pieces.

The same work for Squares and Triangles



Annex V: Examples of the task prepared for student.

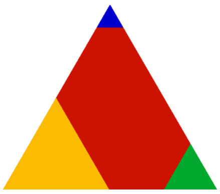
A level

	Erasmus + PUSE Project **Ikastola (school):** … **Grade (A,B,C) /Age: A/6-9** **Task number: A-4** **Time: …**	
Name (student): …		**Date: …**

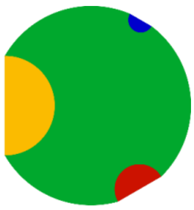
Description: Write the name of the pieces.

For example: S R G B Y

Name: __ __ __ __ __   __ __ __ __ __



Name: __ __ __ __ __   __ __ __ __ __



Name: __ __ __ __ __   __ __ __ __ __

Remarks /Self evaluation:

……………………………………………………………………………………………………

What kind of difficulties have you had? ………………………………………………………………………

………

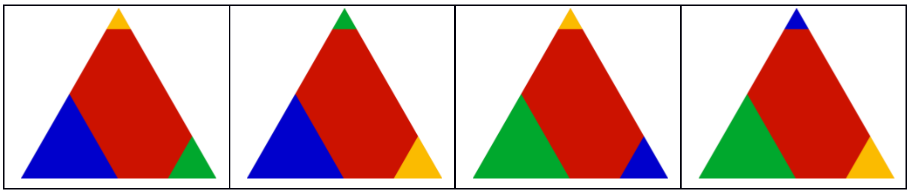
What have you learned doing this activity?

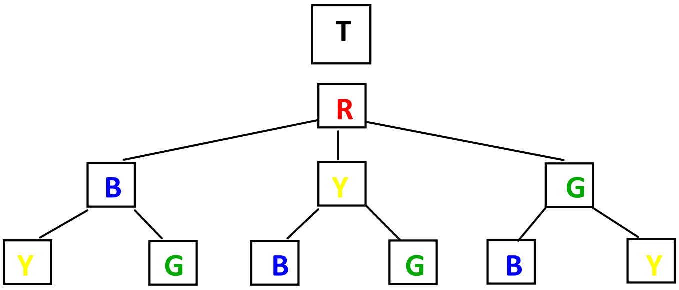
……………………………………………………………………………

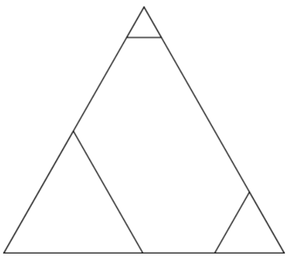
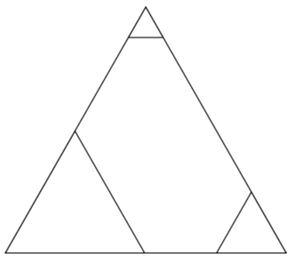
Others:

	Erasmus + PUSE Project **Ikastola (school):** … **Grade (A,B,C) /Age: A/6-9** **Task number: A-5** **Time: …**	
Name (student): …		**Date: …**

Description: There are 6 triangles whose largest part is red. Two are missing. You have to find them, draw them and indicate their name





	
Name: ………………………………	**Name: …………………………………………**

Remarks /Self evaluation:

…………………………………………………………………………………………………

What kind of difficulties have you had? ……………………………………………………………………

……

What have you learned doing this activity?

…………………………………………………………………………

……

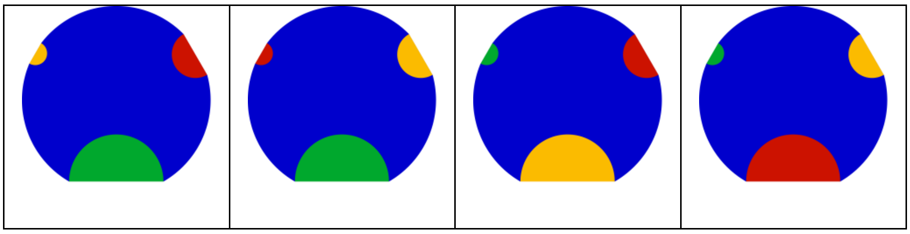
Others:

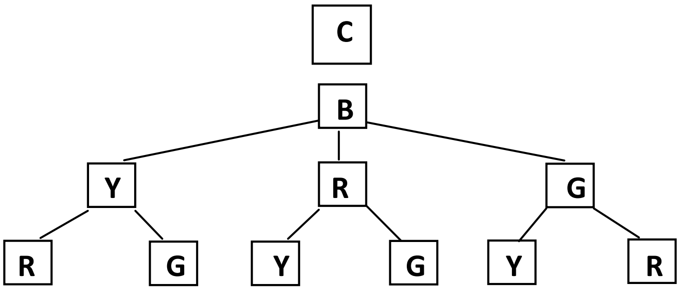
B level:

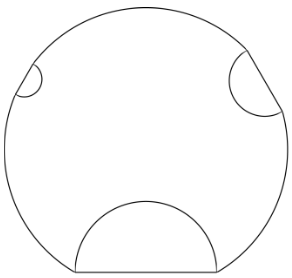
Erasmus + PUSE Project
Ikastola (school): …
Grade (A,B,C) /Age: B/10-14
Task number: B-6
Time: …

Name (student): … | **Date: …**

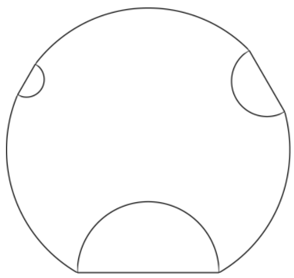
Description: There are 6 circles whose largest part is blue. Two are missing. You have to find them, draw them and indicate their name







Name: ……………………………….



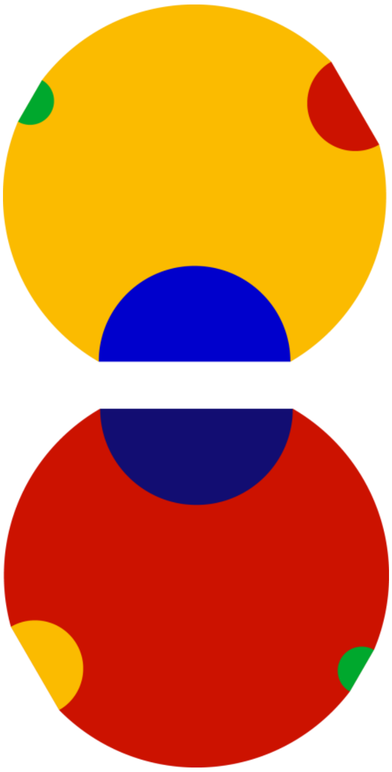
Name: …………………………………………

Remarks /Self evaluation:

…………………………………………………………………………………………………
What kind of difficulties have you had?
…………………………………………………………………………
What have you learned doing this activity?
…………………………………………………………………………………………

	Erasmus + PUSE Project **Ikastola (school)**: … **Grade (A,B,C) /Age: B/10-14** **Task number: B-14** **Time: 10 min**	
Name (student): …		**Date: …**

Description: Look closely at this composition of pieces, cover it or turn the page, and reproduce the composition without consulting it.



How many times have you had to look at it before doing it right at all: ………..

Remarks /Self evaluation:

…………………………………………………………………………………………………

What kind of difficulties have you had?

………………………………………………………………………

……

What have you learned doing this activity?

………………………………………………………………………

……

Others:

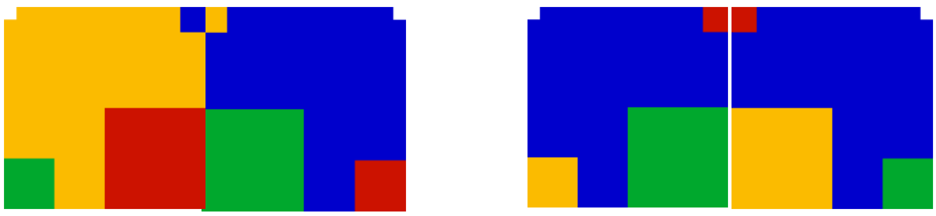
C level

Erasmus + PUSE Project
Task number: c-17
Grade (A,B,C) /Age: c/14-18
Operation: Problem solving
Set: Square
Language:
Time: 15 min

Description: How many differences are between these two pieces? One difference. Only the color of the red and yellow squares

What kind of symmetry can you find in this figure?: Symmetric in shapes and in one color (green)

Complete the chain with all the pieces that you are able to put keeping the rule.

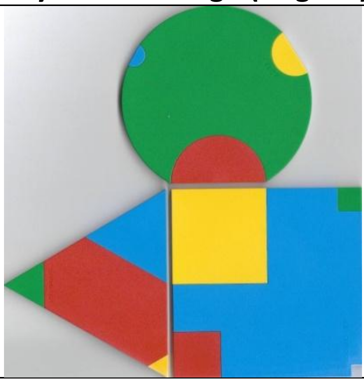
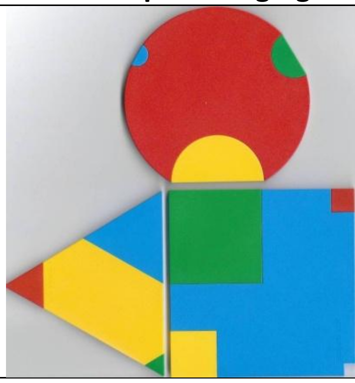


Evaluation of the Task (From 1, less positive, to 4, more positive)	1	2	3	4
Evaluation of the task/ Jardueraren ebaluazioa:				
Adaptation to age /Egokia adinerako:				
Time estimation/ Denboraren estimazioa				
Motivation grade / Motibazio maila:				
Learning value/ Balio didaktikoa:				
Percentage of success/ Arrakasta portzentajea:				
Adequate to the diversity of students/ Egokia ikasleen aniztasunarentzat:				
Favours creativity/ Sormena laguntzen du				
Comments/Oharrak: …				

	Erasmus + PUSE Project **Ikastola (school)**: … **Grade (A,B,C) /Age: C/14-18** **Task number: C-20** **Time: 30 min**	
Name (student): …		**Date: …**

Description: How many ways can the Poly-Universe logo be composed by changing the colors and maintaining the relationship of shape and colors that the parts of the three pieces hold among themselves?

If you need you can consult the annex of figures of pieces.

Poly-Universe logo (original)	**Here an example changing colors**
	

Take pictures of your work.

The number of logos found is: ……..

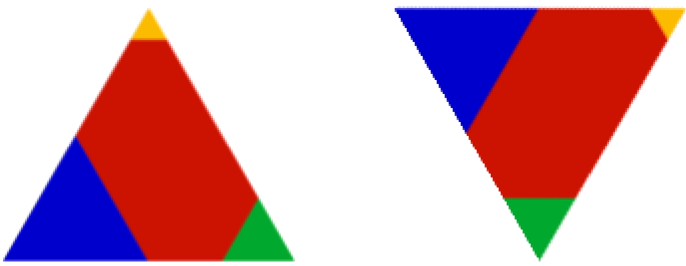
Remarks /Self evaluation:

………………………………………………………………………………………………………

Annex VI :

	Erasmus + PUSE Project **Task number:** **Grade (A,B,C) /Age: B/10-14** **Operation:** **Set:** **Language:** **Time:**	

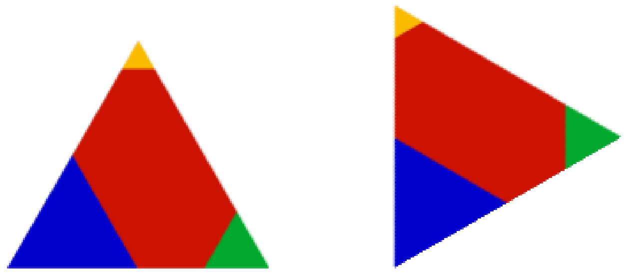
- In the picture you have one piece twice drawn. The piece on the left side is the model, the piece on the right side is the image of the transformation. The question is: what kind of geometric movement/transformation (translation, rotation, symmetry, and groupping) has been done to transform the piece that is on the left side to become the piece that is on the right side? You can see the example.



Kasu honetan translazio bat eta biraketa bat gertatu dira. Biraketa 60ºkoa izan da (ordulariaren orratzen zentzuan)

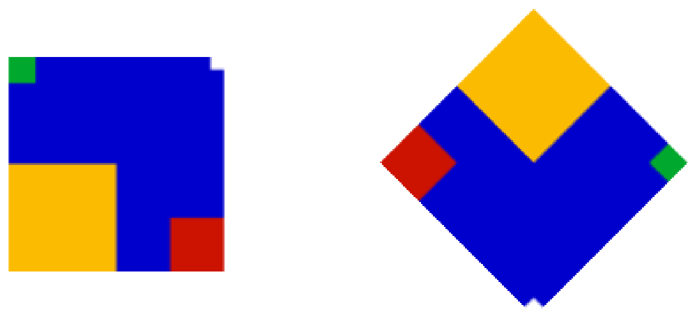
It is your turn. You have to write down the answer in the “Drawing”

1. Exercise. Don´t forget to write down the angle and the direction of the rotation



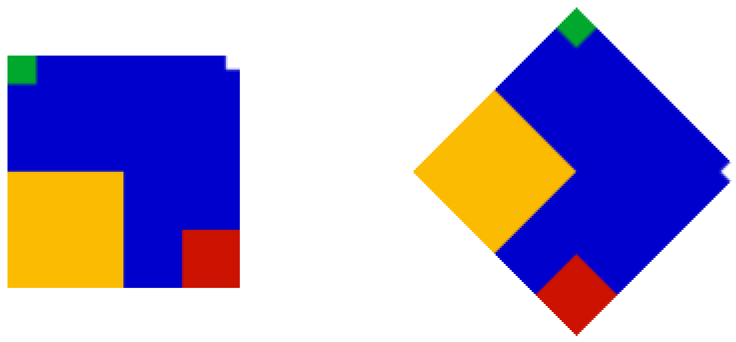
Hemen translazioaz gain ....

2. Exercise. Don´t forget to write down the angle and the direction of the rotation



Hemen translazioaz gain ....

3. Exercise. Don´t forget to write down the angle and the direction of the rotation



Hemen translazioaz gain ………

4. Exercise. Don´t forget to write down the angle and the direction of the rotation

………………………………

PUSE Study

on the use of the Poly-Universe game set

Czuczor Gergely Primary School, Nové Zámky

Základná skola Gergelya Czuczora s vyucovacím jazykom madarským
Czuczor Gergely Alapiskola, G. Czuczora 10, Nové Zámky - Érsekújvár, Slovak
http://www.czuczora.eu
Mónika NOVÁK

Report by

Primary school: **Tibor KOLLÁR**
Upper primary school: **Ildikó MOJZES**
High school: **Gabriel DRÁF**I

The Gergely Czuczor Primary School, the sole provider of education in Hungarian for the Hungarian minority in our town, enables full primary education for 389 students taught by 32 teachers in 18 classes. The teaching staff is supported by other non-teaching staff members including the school psychologist, a special needs teacher and a teaching assistant. We provide education for children aged 6-15 in grades 1-9. Currently there are 2 classes in each grade which means 18 classes altogether.

To characterize the student body, 100% of them declare themselves Hungarian. Approximately 50% of them are male. 25% of the students are commuters from nearby villages; 45 students study abroad. 40 students suffer from some form of learning disability.

Our institution provides the following extracurricular activities: dance class, puppet theatre, ICT class, ancient Hungarian handwriting class, table tennis, English language, preparation for entrance exams to secondary schools in Maths and Hungarian language.

Paying attention to talent development is of high importance for us, proof of which is the fact that our students regularly take part in various competitions, even on national level. In addition, our the to-be journalist students, quarterly issue the school magazine titled Chalk-powder.

Our school has been playing a crucial role in the development and education of Hungarian teachers in Slovakia since 1966. We maintain close partnership cooperation with the University of Constantine the Philosopher in Nitra, Slovakia, as the university's teacher training school.

1. Project phase

The Slovakia-based Gergely Czuczor Elementary School in Nové Zámky is a partner school of the Poly-Universe in School Education (PUSE) project.

In the first project phase (1/October/2017 – 31/December/2017), the heads of the subject committees consulted with the school principal about the project and selected the colleagues who would participate in the project. Three age groups are required for the research, and since there are children aged only 6-15 at our school, we offered cooperation with the Pázmány Péter Secondary School (PPG).

Working with the first age group was undertaken by Tibor Kollár. He is the head teacher of the 3rd graders, which is the reason why his class was to become the pilot group and the parallel class would be the control group.

The second age group was taken by Ildikó Mojzes, who has started working for our school this academic year. She teaches Math in class 5B and they became the pilot group, while the parallel class 5A is the control group.

The third age group was the tenth-grader clasas of PPG. Since Ildikó Mojzes had been teaching there up until recently and there is excellent cooperation between the two schools, the solution was to divide the large number of tenth-graders (32) into a pilot group and a control group. Their work is supported by a colleague of theirs, Gábor Dráfi, who as an English teacher colleague coordinates linguistic issues.

All three of them attended the first transnational project meeting in Budapest, December 2017.

In the second project phase (1/January/2018 - 31/May/2018) we implemented the PUSE measurements with the selected classes and teachers in the initial, middle and final stages. The essence of our work was to test the tasks selected from the tasks bank and, naturally, to create, try out and upload new tasks to the tasks bank.

2. Project phase

The 1-4 graders measurement

In January, we started to select classes in the lower and upper grades. At our school 15 students from class 3.A have become the control group and 18 pupils from class 3.B the pilot group. These students received a paper-pencil-based test (task sheet), which was used to measure the basic skills of attention and perception separately, and which had to be completed within a given time limit. Their results were sent off in an aggregated spreadsheet. The first PUSE measurements in both classes in the third grade took place on 25 January 2018. The number of students measured was due to the absence of 5-6 students in each class on the day indicated.

From the beginning of February work began with the tools. An average of one hour per week was included in the timetable on which we worked with the tool. This work lasted until the end of May. First, we introduced the three different sets of the Poly-Universe to the students. In the following meetings we performed various tasks, made observations, recorded notes, attempted to create similar tasks, searched for and collected the possibilities and tasks that could be related to the tool and mathematics, made photos of the completed works and solutions. The second assessiment of the classes took place on 7 June. The same pencil paper test was written under the same conditions as in January. Due to absences in class B we were able to measure 3 pupils less than before. All 15 students from class A who first participated in the measurement could be measured. The results of the data were sent to psychologists for evaluation. We are looking forward to the analyses that the students involved in the assessment are very curious about.

The measurement of 5-9-graders and Pázmány Péter Secondary School students

In January we looked at the curricula and tried to select topics that we could attach Poly-Universe tasks to in all three age groups.

We first implemented the PUSE measurement. Unfortunately, with the 3rd age group it was difficult to start the measurement as we could only work with them outside the curriculum at the so-called zero lessons on Tuesday and Friday mornings. Their web system could not handle the load, so we had to do the measurement in the IT room of the Gergely Czuczor Elementary School, which was a logistical challenge.

We worked with each of the groups in the frequency of one to two hours a week. Work was most often done in co-operative groups but of course, depending on the tasks, pairwork and individual work also occurred.

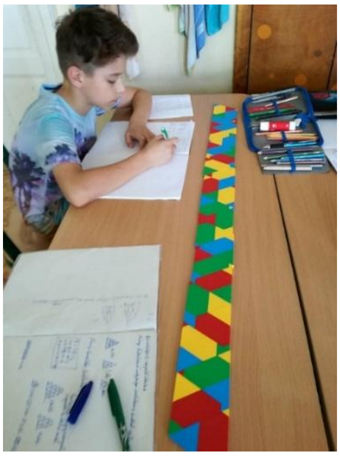
We constantly monitored the students' work, prepared worksheets for them and recorded their solutions on photos.

We uploaded the original task ideas along with photos into Google Drive.

Ildikó Mojzes and Gábor Dráfi prepared the report for the 2nd project meeting in the form of a presentation.

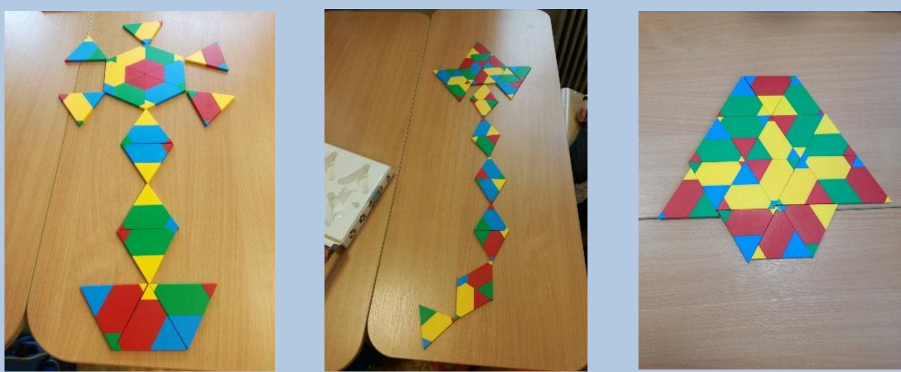
The Experience Workshop organized the 2nd transnational project meeting in Jyväskylä, Finland. The purpose of the meeting was to prepare the teaching methodology. The partner institutions presented their achievements. We also visited two Finnish schools. During our short stay we also gained some insight into the Finnish educational system.

The meeting emphasized the Poly-Universe methodology and the form of the published workbook.



Example tasks

When children saw the set for the first time they were extremely creative. They had surprisingly new vision, but in group C the students' imagination seemed to be much more difficult to bring about.



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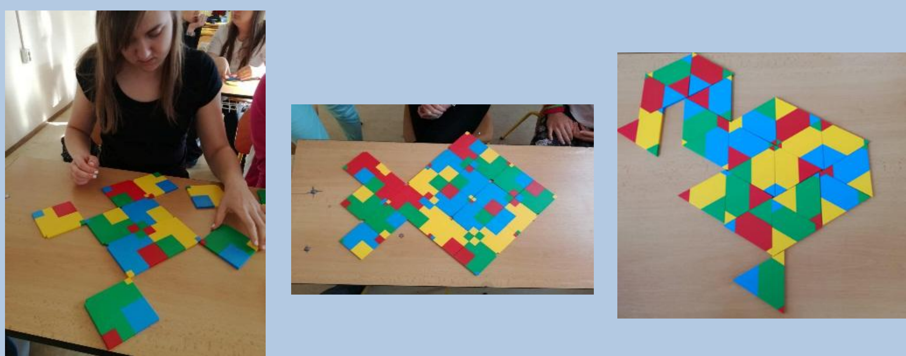
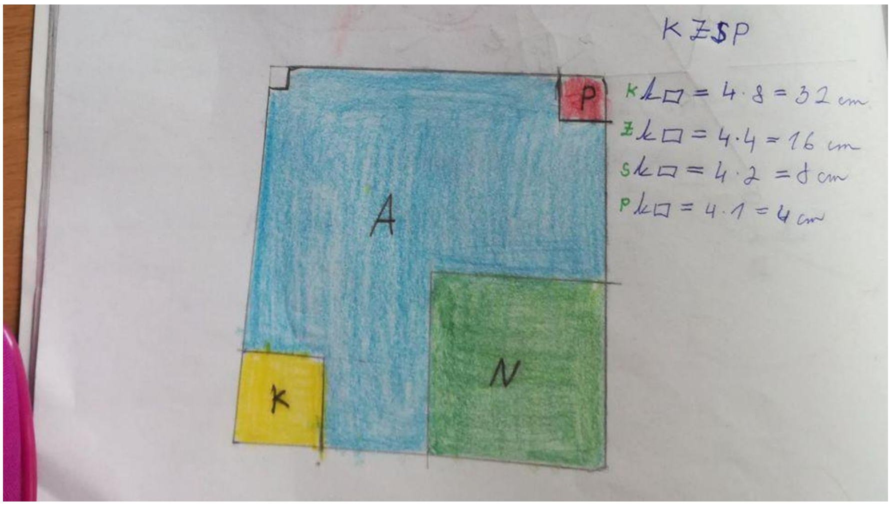


C

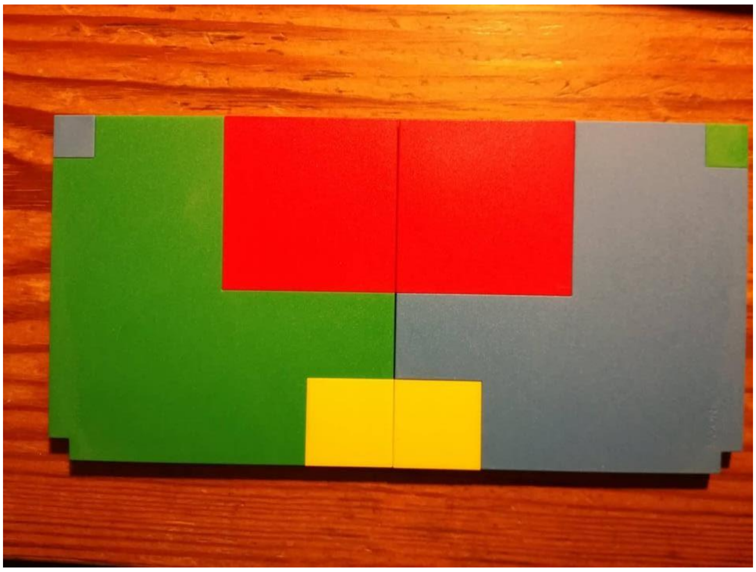
Redraw any piece of the square set using a ruler and a pair of compasses into your notebook. What is the name of your piece based on the coding we made up? Calculate the perimeter of all the rectangles.

This task was created for age B. It turned out to be very helpful for the practice of constructing parallel and perpendicular lines.

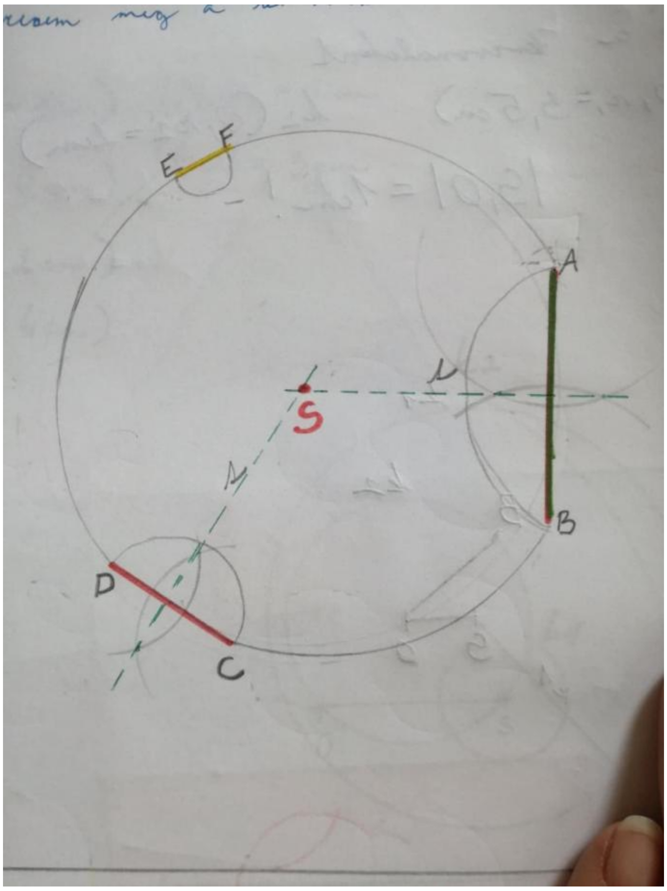


Place two pieces next to each other so that two more identical colour rectangles will be created. Draw them into your notebook. Calculate the perimeter of all the rectangles.

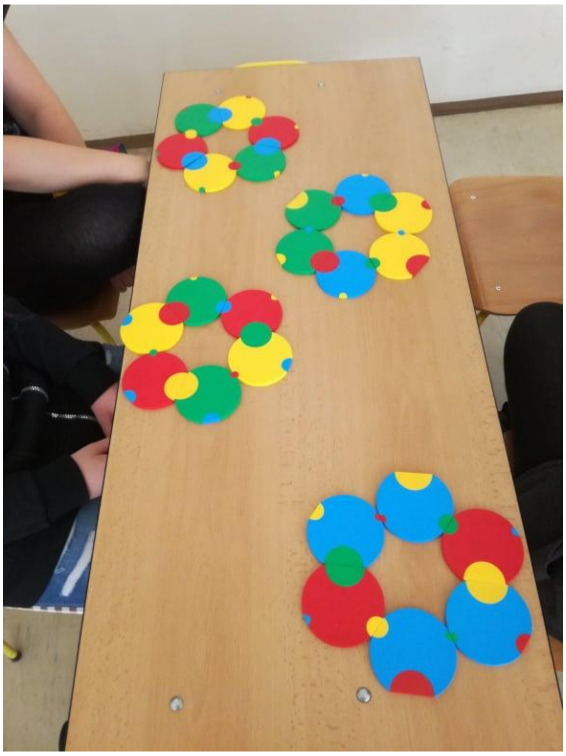
This task was created for groups A and B. While group B practiced calculating perimeter only, group A had another opportunity for practicing the construction of parallel and perpendicular lines.



Find the centre of the base circle. Use any piece of the set. This task was solved by both groups B and C, but on different levels. Axis of symmetry was unknown for group B, so first it was introduced to the students. Group C was led to the right solution by asking them leading questions.



Connect 6 pieces in full circle so that the connected parts are identical in colour. Work in groups. Can you put together the whole set if each of your form a flower?



Create isosceles trapeziums of different size. How many triangles do you need to use to make the smallest isosceles trapezium? How many triangles do you need to make the biggest possible isosceles trapezium?

At the previous lesson 3 different quadrilaterals had been introduced to the students which are possible to create using the set. Working in pairs, they put together the smallest isosceles trapezium from three parts and they also managed to create the biggest one. Later they had similar tasks for creating isosceles triangles and rhombuses.

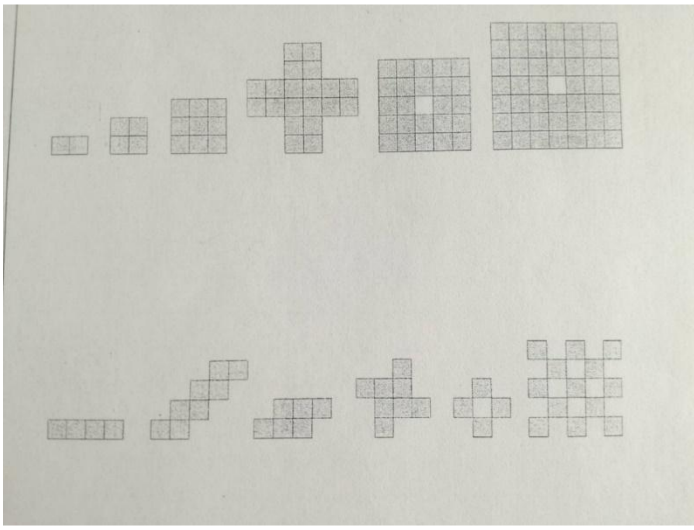


Freely put together some pieces of the circle and the triangle set. Guess what I have created. What can you see?

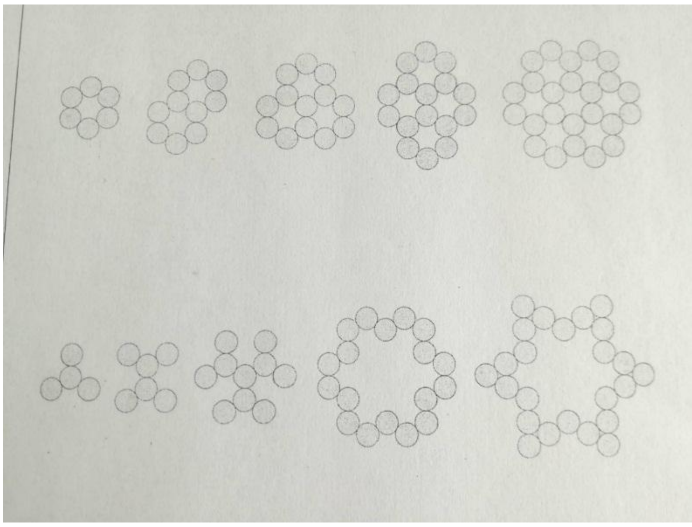
While group B was working with the circle and triangle sets, group C was working with the square and circle sets. Since they had used the sets before on many lessons, they consciously used the rules they had made up in groups.

Put down 4 pieces from the square set so that everyone has a different size but the same colour square. What rule have you found out about the perimeter?

As members of cooperative groups they had to support each other to form the shapes in the task using the set. They also managed to find out a rule for calculating the particular perimeters.




Make closed shapes or open shapes from the circle set in groups based on the picture. Choose the rules yourselves.





Group C created a shape which they the taks did not include. They created a shape which was partly open, partly closed.


Feedback from students in Phase 2.

I will need your help for the next part of the project. Please take the task seriously and answer the question below.

1. How did you feel yourself at the POLY-UNIVERSE lessons?
   Positive answers: *Supergood! I was looking forward to it! They were very exciting.*
   They were not any negative answers.

2. What did you like or didn't like when you worked in groups?
   positive answers: *we could help each other, everybody was working, I could be with my friends in a group. Everybody could say their opinion. We listened to each other.*
   negative answers: *Sometimes they were selfish and did nothing.*

3. Which set did you like the best? / 80% said triangles.

4. What was the most interesting task?
   When we could freely create, when we made shapes based on a picture.

5. Would you like to use the set on your lessons in the next school year?
   95 % said yes, very much

Poly-Universe

– Results of the PUSE project impact assessment –

Authors: Tokai Dalma, Damsa Andrei, Dr. Fromann Richárd
NetCoGame JátékosLét Research Center

1. Introduction

The research was conducted within the **Poly-Universe in School Education** (PUSE) ERASMUS+ project. The Polyuniversum (PolyUni as a toy and a development tool) studies' goal was to get an insight of the positive effects of the activities from cognitive, social, and emotional perspectives related to the time spent with the tool. To gather this information, we designed a study involving different countries, methods, and age groups. The development programs were created to respond to local and personal needs of the children.

As presented by many classical psychological studies, games and the act of playing can fulfill an important role in the development of the children and also of the adults. This study is equally based on validated, already acknowledged cognitive psychological measurements, attitude exploration, and subjective and objective experiences of the professionals working with the PolyUni method. The gathered information and the extracted conclusions are reflecting the student's and educator's opinion on the PUSE methodology and its effect on the development of different areas.

Structure of the study:

1. Introduction, the importance of game research
2. Introduction of the PUSE study
3. Presentation of the research, data analysis and interpretation
4. Conclusion

In the introduction of the study a theoretical background is provided for the research. After this part, we will examine the PUSE methodology from a more practical and life-like aspect, focusing on the developing psychological areas. By analysing the gathered data we have the opportunity to form scientific statements about the usability and validity of the development tool.

1. 1 Game research

Many of the game-related research projects' goal is to define their positive effects, and related to this, the working methodologies behind playing. Besides, in most cases researchers focus on digital games (Fromann, 2017), the analogous or physical games can also produce interesting and exciting results. This is fortified by the fact that these kind of games are becoming more and more popular again.

The relevance of game research, linked to the PUSE study, is based on psychological and sociological measurements. Progression associated with playing can be approached from two different angles. In 1978, Lev Vygotsky draw attention to the importance of games in the context of child development. He stated that playing is more than spending free time with aimless activities. In the researcher's theory children go behind they mental age and explore new fields of knowledge. Huizinga (1949) formed very similar statements from a different point of view. The Dutch anthropologist examined games and playing in the context of smaller and larger groups. He stated that games are essential parts of forming communities and creating culture. Through playing, groups get a sense of becoming a team and can form common knowledge and beliefs.

These two directions can show us that games can be useful tools in the examination of human nature. Based on this, and other related fields (Damsa, 2014), we believe that the research of development games, just like PolyUni is highly recommended and senseful.

2. Measurements

2.1.Poly-Universe - about the game

Poly-Universe is basically a developing game consisting of geometric forms. In the game we can operate with 3 types of forms: triangles, circles and squares. The elements consist of different colours: blue, yellow, green and red (Figure 1.)

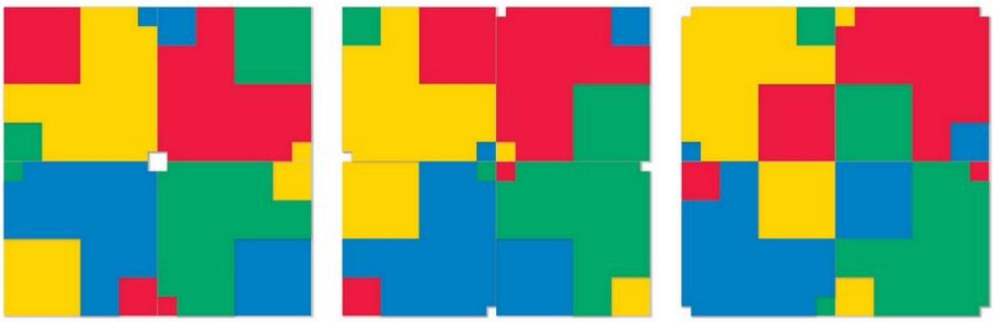


Figure 1. Examples from the Poly-Universe

From the four colours, there are three different ones in each element - the Poly-Universe developmental pedagogical methodology is based on these elements. The game consists these three 'families':

- equilateral triangle
- circle
- square

Each set is made up of 24 elements, every single one of them is unique. Elements can be matched by different rules, for example by colour or by size.

2.1.1. Building in the game

Poly-Universe is a source of common experiences, recognitions, means of joyful creating. The game creates direct contact between the players and the geometric forms: students can hold the forms in their hand, they solve tasks, they have the opportunity to recognize patterns, rules, regularities. Also, it moves their imagination. Behind the simple geometric form there is a big complexity and many-many varieties, its power is its strength. It can be useful and interesting for each age group; the mathematically talented or the ones dealing with difficulties, both can benefit from the game. The great number of possibilities are a continuous challenge for the students.

There are no strict, exclusive rules for the game. We can make students face various problems, tasks to be solved. During the sessions they automatically find the aesthetic, mathematical regularities, which can be the following:

a) Discovering geometrical forms
b) Discovering ratios, proportions
c) Finding symmetries
d) Finding connection points
e) Defining directions
f) Connecting colours
g) Mixing forms
h) Expanding compositions
i) Finding all the possible combinations
j) Experiencing the infinite

2. 2. Areas of the research

Our presupposition was that working with PoliUni on a regular basis can help developing many cognitive abilities. It requires concentration and planning; during the solving of the tasks it helps to keep the forms in mind. What have you already used, what is that you haven't? We hypothesized that regular usage of the game may help to develop short-term memory.

Furthermore, we think that abstract thinking as well as the order of mental operations can be developed by the game. Mixing and matching of the elements, planning of the next matches requires these skills. To create big, complex forms from the elementsplanning ahead is required. We aimed at grabbing and operationalizing this abstract thinking, and we hypothesized that this kind of thinking is developed by the regular use of Poly-Universe.

Not only the abstract, but the concrete thinking may also be developed by the game. It develops geometric abilities mainly in a direct way. We think it's important to measure subjective factors besides the objective ones. We suppose that besides the skills development, it helps to increase learning motivation, attitude towards school and mathematics. During the sessions with PoliUni, they think of mathematics and learning as a game, they learn by playing.

These sessions are less restricted by rules than in a classical mathematics lesson. They can set their imagination free, they can realize that the things they learn in math class can be transformed and used in many aspects of life.

Amongst the skills of students and their affective factors (attitudes towards school and mathematics) we wanted to examine the effect of PoliUni from the aspect of the teachers. How do they see the effect of the game? What do they think, what are the strengths and possible weaknesses of the game? We wanted to get feedback from teachers on these topics.

The aim of our project is to increase the efficiency of teaching mathematics by building the PoliUni into the urriculum. We want to make mathematics useful, we want to make the students see that math is useful, we want to increase learning motivation and form the attitude towards school for the better. Our aim is to let the students see that learning can be fun and playful.

By measuring the skills and attitudes of students and opinions of teachers, we can get a complete picture of the effect of PoliUni, making the game as useful as it can be as part of the curriculum.

Our hypotheses:

1. We predict that by using PoliUni, the attention span increases, so a group of children using PoliUni on a regular basis is going to perform better in an attention test than the control group.
2. We predict that PoliUni helps developing visual perception, so students using PoliUni on a regular basis are going to perform significantly better in the Embedded Figure Test than the control group.
3. Due to the inner planning and playing by the rules, we predict that students using PoliUni are going to perform statistically significantly better than the control group in the short term memory test.
4. We predict that due to the operations with the geometric forms, students using PoliUni are going to perform statistically significantly better than the control group in the Mental Rotation tests.
5. We would like to explore the accidental changes in attitudes towards mathematics and school, we did not have a presumption in this area.
6. We explored the feedback of teachers about PoliUni without any hypotheses, we wanted to know their opinion, attitude, criticism towards the game.

In the next chapter we are going to display our sample, our methods; we inspect our hypotheses with the help of statistical analysis.

2.3. Sample

We did our survey in each country our project is running:

- Hungarian
- Finnish
- Spanish and
- Slovakian schools.

We had schools where we organized regular PoliUni activities for students in each of these countries.

We had 3 cohorts in each schools from each countries: 7 year-olds (second graders), 11 year-olds (5th graders) and 14 year-olds (8th graders). Our youngest and secound youngest age group follows Piaget's developmental theory. He specified different stages of the cognitive development: the first (sensorimotor stage) is approximately the first 2 years of the child's life. The second (preoperational) stage is from 2 years to about 7 years; our youngest age group fits this. The third (concrete operational) stage ends around 12 years, our second youngest age group fits this. The last stage is the formal operational stage, starting from 12 years, our last age group fits this. The operation itself is a reversible, information-transforming mental rule. They are integrative, which means the stages are built on one another, one is needed to get to the next. In the concrete operational stage the egocentrism ceases to exist, which until then has ruled the child's thinking. Until this stage, the child could take into account only one of an object's qualities (Piaget, 1978). In the concrete operational stage thinking becomes systematic, the child can operate, tabulate, but cannot handle complex systems yet. In the concrete operational stage the child understands that the qualities of the objects are permanent.

In the formal operational stage the child can think abstract, combinatorial ways (Piaget, 1969). Children realise that different qualities of an object are independent from each other. Now thinking is abstract, the child can deal with possible combinations, not only objects (Piaget, 1978).

We chose our age groups along these stages of the cognitive development. We had altogether 267 students, the groups are visible in Table 1.

Age group	Number	Mean age (years)	Standard deviation of age (years)
I.	131	7,9	1,6
II.	76	11,4	0,6
III.	60	14,6	0,6
SUM	267		

Table 1. Dispersion of the participants

The gender distribution in the second and third age group was balanced, both groups had 49% of boys and 51% of girls.

2.4. Methods

We searched through the literature to find the most adequate methods to measure. We present the methods along the age groups; we measured them two ways to be valid and reliable. In the first age group we measured attention, in the second and third age group we measured visual perception, short term memory, mental rotation and learning attitudes.

2.4.1. First age group

Due to differences in cognitive abilities, with our youngest group we had to use different measurements than with the other two.

We used the Pieron-test to measure thinking and attention. In this test the task of a child is to find some specified objects in a sheet filled with objects. This means 2 objects (chair and lamp) from 6 kinds of objects, in our case. The child has 5 minutes to find each object on the sheet.

The assessment happens through 2 dimensions:

1. How many object did the child find, how many did they missed?
2. How fast do they work, how many objects are found in each minute?

2.4.2. Second and third age group

To measure short-term memory we used the so-called *number-spread test.* According to George Miller's (1956) results, the span of the short-term memory is about 7±2 items. This is the number of the items a can be stored in short-term memory. One of the components of this short-term memory is the phonological storage, this is responsible for keeping the information, in a lower level of cognition. This practically means inner repeating, without any deeper processing. This fades away after 1,5-2 seconds and remains unreachable. The other element of short term memory is an articulation control process, based on inner speech, this is what refreshes the informations in the phonological loop (Baddeley, 2001).

To measure the phonological loop the most popular method is the number-spread test. The teacher reads a row of random numbers to the student, who has to repeat them in the same order as heard. The row starts with small numbers, mostly 3, and if the student is able to repeat them, it enlarges by 1 number. The enlargement continues as long as the student makes an error. The largest number of repeated items makes the range of the student's short-term memory (Németh, 2002). Vajda Zsuzsanna writes about the capacity of short-term memory: at kindergarten age a kid can store about 2-3 items, a five year-old can store about 4, a six year-old can store about 5, and about 10 years the development ends and reaches the 7±2 specific to adults. There is a turning point at the age of 7, when they can learn how to operate with their memories, not only store information in them. This operational aspect development ends around 12 years (Vajda, 1999).

To measure cognitive, perceptual skills we used the Embedded Figures Test (Oláh, 1987). The test measures the ability to process and restructure the stimuli coming from the environment. The test is a perceptual challenge, its goal is to find a simple geometric form in a more complex one, when the subject cannot see both figures at once. Simple forms comen at first, then more complex ones after it (1987).

We measured abstract thinking with the Mental Rotation Test. In the test the subject has to decide which of the 4 forms is the same as the reference form. The options are all shaped a bit similar, so solving the task requires mental computation. In most cases, only one option is correct, that is, exactly the same as the reference. However, it happens that there is more than one correct answers. The test measures students' ability to recognize a given form in a shaped manner, potentially in various ways.

We measured school and mathematical motivation, attitudes with short, open-ended questions. We asked them 6 questions:

1. How much do you like your school?
2. If you had to change schools, to what extent would you miss your current school?
3. In your opinion, how much better or worse your school is than other schools in general?

4. How much do you like mathematics compared to other subjects?
5. How useful do you think are the things you learn in math class?
6. How possible is it that you are going to choose a profession related to mathematics?

The students had to mark their answers in a Likert-scale from 1 to 6, where 1 means "Not at all", and 6 means "Totally".

We predict that regular use of PoliUni makes learning fun, a good experience, so the motivation of students increases with its regular use. We predict that the game makes them more willing to take part in classes, to be more active, their motivation and love for mathematics grows. They start to realize that the things they learn in math class are useful in many aspects of life.

Our subjects took the tests twice: at the beginning of the project and at the end. The time between the two measures was around 5 months. The first was in January, 2018, the second is at the end of the school year, June 2018. We had control groups as well for each age group.

3. Processing and interpreting the received data

The interpretation of the received data has followed the classic statistical data processing system. As it was mentioned, the data recording from the first age group was done on paper, while in the case of the second and third age group we used online methods. Despite the fact that the online data gathering created structured data tables, it was inevitable for us to filter the data and delete the inadequate recordings.

The analysis process has followed these steps:

1) Drafting research questions and assumptions
2) Definition of data recording tools and methodology
3) Testing
4) Collecting data tables, sorting
5) Assembling codebooks of the variables, creating a variable list
6) Data cleaning, noise elimination
7) Systematization of the cleaned tables, assembling a unified database
8) Variable computation, assembling groups and recapitulative variables
9) Running descriptive statistics
10) Running statistical tests related to assumptions and research questions
11) Validating research results, secondary check

12) Summarizing research results, making interpretations

This chapter summarises the data processing and focuses on the results extracted from the databases, along the assumptions mentioned previously.

According to the the first hypothesis, we presume that the attention capacity grows because of the impact caused by the PoliUni, therefore the members of the experiment group have earned statistically higher score on the chair-lamp test than the members of the control group.

3.1 Examination of attention

Regarding the effects of PUSE activity on attention, a chair-lamp test was created. The reason for the test is to reveal the rate of students who can seclude the target-stimulus from a stimulus-palette fortified with noise. For comparing the results of the control- and the experiment group, firstly we checked the dispersion of the scores. We saw two normal dispersions, using the Shapiro-Wilk test, so for the comparison of the group results we had to use paired and independent Student t-test. The results of the normality examination test is summarized in the Table 2.

Group	W - value	Significance value
Control - premeasure	0,8	$p < 0,05$
Control - postmeasure	0,7	$p < 0,05$
Experimental - Premeasure	0,6	$p < 0,05$
Experimental - Postmeasure	0,7	$p < 0,05$

Table 2. Results of the Shapiro - Wilk normality test

Comparing the results of pre- and post-tests, we got statistically significant differences between the two groups ($t = -2.5$, $df = 40$, $p < 0,05$) , therefore we can say that attention skills with or without PUSE activity are significantly developing in the group of the participants. Similar but stronger effects can be observed in the experiment group, where the attention capacity grows, too ($t = -3.8006$, $df = 78$, $p < 0,05$).

Regarding the received results, we can say that during the PUSE activity, the scores of the attention-skills increased, however we can't ignore the fact that we can find this growth at the members of the control group as well. From the age-specific characteristics of the target group, we can assume that the attention-capacity develops.

3.2 Examination of visual perception

The research of the second hypothesis is focused on the development of the visual perception brought by PUSE. The assumption says that those kids who participated in PoliUni activities got significantly higher scores on the embedded figure test compared to those in the control group. To reveal the differences between groups we used paired and independent t-test. The criterion of the Student t-test is that the pattern have to follow a normal dispersion, and because of that we used Shapiro – Wilk normality test. During the test, we diagnosed that the patterns containing pre- and post-measures follow normal dispersion, so for comparing the variance of groups we used Student t-test. The results of the Shapiro - Wilk test which confirms the normal dispersion are shown in Table 3.

Group	W-value	Significance value
Control - premeasure	0,5	$p < 0,05$
Control - postmeasure	0,95	$p < 0,05$
Experimental - premeasure	0,95	$p < 0,05$
Experimental - postmeasure	0,94	$p < 0,05$

Table 3. Results of the normal dispersion examining Shapiro-Wilk test

The comparison of the groups follows this pattern:

- Experimental group first measurement / Control group first measurement
- Control group first measurement / Control group second measurement
- Experimental group first measurement/ Experimental group second measurement
- Experimental group second measurement / Control group second measurement

First, we examined the potential differences between experimental and control group with checking the basic nature of the data. The condition of the examination is the starting equality between groups. Regarding the use of independent patterned t-test to compare the groups, it didn’t showed significant difference in the pre-measurement in the case of the embedded figure test ($t = -0,048$, $df = 117,61$, $p > 0,05$). In conclusion, we can confirm that there were no major differences between the control and the experimental group. To check the effects of intervention, we did post-measures in the case of experimental and control groups as well.Regarding the comparison of the two groups, we can say that unlike the results of the pre-measure, in this case there is a strong tendency for the experimental group (t = -1,807, df = 116,11, p = 0.07).

We can confirm that after examining the differences between pre- and post-measures, there were no different results between the two data recording times (t = -1.3981, df = 57, p > 0,05). Based on this we can say that in the case of the control group the passing of the time did not resulted in major development in visual perception. However, in the case of the experimental group we can confirm that they were better at the time of the second measurement than at the first time (t = -4.4927, df = 77, p < 0.01). The averages, received t-values, the degrees of freedom, and the p-values are summarized in Table 4.

group/comparison	Average	t value	Degree of fredom	p value
Control pre-measurement / Experimental premeasurement	4,6 / 4,6	-0,048	117,61	p > 0,05
Control pre-measurement / Control post-measurement	4,6 / 4,9	-1.3981	57	p > 0,05
Control post-measurement / Experimental post-measurement	4,9 / 5,5	-1,807	116,11	p = 0.07
Control pre-measurement / Experimental post-measurement	4,6 / 5,5	-4.4927	77	p < 0.01

Table 4. Comparison of variances of the group results earned on the embedded figure test

Based on the received results we can confirm that students who participated in PUSE activities have earned significantly better scores on the embedded figure test than those who did not participated. Concerning the measures, we can say that gaming with PoliUni develops skills related to visual perception, measured in the embedded figure test.

3.3 Examination of short time memory

While examining the third hypothesis, we wanted to find out how PUSE activities affect the short-term memory. We assume that PoliUni participants get significantly higher score on number memory test than the control group members in complying with the rules and internal planning processes. The examination was carried out by the mentioned number memory test, the research design is similar to the layout of the embedded figure test.

The revealing of that data was started with dispersion pattern examination which showed that in some cases the density function of the values are not following the normal dispersion, thus we used the Kolmogorov – Smirnov test. The distributions are summarized in Table 5.

Group	W - value	Significance value
Control – pre-measurement	0,97	p > 0,05
Control – post-measurement	0,88	p < 0,05
Experimental – pre-measurement	0,96	p < 0,05
Experimental – post-measurement	0,96	p < 0,05

Table 5. Results of the Shapiro-Wilk test, examining normal distribution

During the testing of the hypothesis, we compared the pre- and post-measures of the control group, and as in this case the frequency curve doesn't followed the normal dispersion, we used Kolmogorov – Smirnov test. The received results show that there is difference between the two measures ($D = 0,258$, $p = 0,04$, averages: 12,7 / 12,1) without intervention from the PUSE. Starting from the fact that in the case of the control group the average decreased in the second measure, a comparison with the experimental group can be applied. During the premeasure of the experimental and the control group, there were no major difference between the two patterns ($D = 0,104$, $p > 0,05$). During the examination of the pre- and post-measures we didn't find significant difference either ($t = 1,507$, $df = 77$, $p > 0,05$). In conclusion, we can confirm that PUSE activity doesn't affect the developing of number memory and short-term memory in the case of the examined pattern.

Comparing the scores in the experimental and control group post-measures, there is no statistical significant difference either ($t = -0,2204$, $df = 100,8$, $p > 0,05$). The summarized values are shown in Table 6.

Group/comparison	Average	Values	Degree of liberty	p value
Control premeasure / Experimental post measure	12,7 / 12,8	D = 0,104	-	p > 0,05
Control premeasure / Control post measure	12,7 / 12,1	D = 0,258	-	p > 0,05
Control postmeasure / Experimental post measure	12,1 / 11,8	t = - 0,220	100,8	p > 0,05
Experimental premeasure / Experimental postmeasure	12,7 / 11,8	t = 1,507	77	p > 0,05

Table 6. Comparison of variances of the group results, earned on the number-memory test.

3.4 Examination of mental rotation

During the cognitive examinations of PUSE activities, the abstract operation with geometrical shapes had priority, which is closely linked to PolyUni's methodology. Students participating in research did the mental rotation test before and after the activities. Similarly to the scopes presented before, we also monitored the performance of the control group.

For choosing the adequate statistical test, we examined the score-distribution in the groups. Using the Shapiro-Wilk test the result is normal dispersion in each case. Starting from this, we used independent and paired patterned t-test to compare the groups. The results are shown in Table 7.

Group	W-value	Significance value
Control - pre-measure	0,93	$p < 0,05$
Control - post-measure	0,96	$p = 0,05$
Experimental - pre-measure	0,91	$p < 0,05$
Experimental - post-measure	0,95	$p < 0,05$

Table 7. Result of the Shapiro-Wilk test, examining normal dispersion

For the comparison of the control and experimental group, we had to reveal the groups' initial position relations to each other. Using independent t-test it became visible that statistically there are no significant result from the pre-measure (t = -1.4841, df = 132.81, p > 0,05). The comparison of the pre- and post-measures of the control group give similar results, there were no relevant changes concerning the time during the mental rotation test (t = -1.9117, df = 57, p > 0,05). Scores of the experimental and control group earned on the post-measures do not show statistically significant differences. The summary of the values is shown in Table 8.

Group/comparison	Average	t-value	Degree of freedom	p value
Control premeasure/ experimental premeasure	3,8 / 4,1	-1.484	57	p > 0,05
Control postmeasure/ experimental postmeasure	3,8 / 4,3	-1.618	132.81	p > 0,05
Control postmeasure/ experimental postmeasure	4,3 / 4,2	-0,220	100,8	p > 0,05
Control premeasure/ experimental postmeasure	4,1 / 4,2	-0.360	77	p > 0,05

Table 8. Comparison of variancies of group results in the number memory test.

Based on received results we can confirm that PUSE activities do not significantly affect the mental rotation skill measure test scores.

3.5 Examining the attitude of students

When recording cognitive tests, we focused on examining how PUSE activities affect students' attitude, for example: are beliefs about school or mathematics affected while playing with the PolyUni game. During the test we asked five questions from the students, which were the following:

1) How much do you like your school?
2) If you had to change school, how much would you miss your current school?

3) To what extent is your school better compared to other schools?
4) How much do you like mathematics compared to other subjects?
5) How useful do you find things learned in mathematics classes?
6) Is it possible that you will choose math-related subjects in the future?

The students could answer the questions on a Likert scale from 1 to 6, depending on their negative or positive opinion (1 – negative, 6 - positive). Because of the subjective nature of attitudes we summarized the averages in relation to each other. The comparison is shown in Table 9.

	Control group		Experimental group	
Question	Pre-measurement average	Post-measurement average	Pre-measurement average	Post-measurement average
How much do you like your school?	4.637931	4.465517	4.128205	4.272727
If you had to change your school, how much would you miss your current school?	4.672414	4.551724	4.584416	4.519481
What do you think, to what extent is your school better than other schools?	4.517241	4.448276	4.974359	4.545455
How much do you like mathematics compared to other subjects?	4.087719	3.758621	4.089744	3.545455

How useful do you find the things learned in math class?	4.758621	4.473684	5.012821	4.415584
How likely is that you continue your studies on a field related to mathematics?	3.827586	3.684211	3.662338	3.714286

Table 9. Summary of the averages of attitudes measurement among students

(1 – negative, 6 – positive pole).

As can be seen from the table above, there is no outstanding difference between the averages. The reason for this may be that the time for the task probably wasn't enough to make visible differences. Furthermore, there is an important role in the different impulses which influence the student's answers in multiple ways. Based on the attitude experiment we can say that during this measurement there was no influence/effect which can be related to the activity of PUSE.

3.6 Attitudes and experiences amongst educators and developers

Besides the participating students in PUSE activity, we find it important to ask questions from educators and professionals who are working on the development or on activities with the equipment/tool. Therefore, we used an online questionnaire which contained questions based on Likert-scale and open questions. The goal was to get an insight in the effect of the PoliUni methodology through the subjective opinion of the professionals on these fields:

- Creativity
- Mathematical and logical skills
- Social skills
- Attention

Research on these fields covers the amount and the nature of the progress granted by the professionals. The questionnaire was filled by 14 educators and professionals from the following countries:

- Hungary
- Slovakia

- Spain
- Finland

3.6.1 Creativity

On the examination of creativity, the participants answered the questions on a Likert-scale from 1 to 6 about their opinion of the extent PUSE can develop creative thinking. During the processing of data the average score was 4.4, the deviation 0.9. According to this, the educators and professionals believe that the PUSE tasks develop creativity and creative thinking.

This is supported by the following statements from the questionnaire, formed by the participants:

"It motivates the children, they like to play with the set, vary the elements, enjoy the adaptability and polychrome of the forms."

"From the given set, with given parts they try to create the biggest possible formation, so they can see many opportunities in a task."

"The problem solving skills of students develop; it was a new for most of them that they can use their mathematical knowledge in a different context than usual; the task with the Poly-Universe was a game, actually like a "reward", through which they can have a break in math class."

"Because there are no rules, everybody makes them by their knowledge. So the player can deepen the exploration of connections between forms and colours freely, without boundaries."

"When they meet the game, they show signs of curiosity, waiting, sometimes resistance…, but when they solve a logic, mathematical problem, they get excited and experience the "heureka" feeling."

As professionals outlined, the PolyUni was a popular game among the students, during which they solved creative problems, without feeling the pressure of performance.

3.6.2 Mathematical and logical thinking

The second field of research focused on the mathematical and logical thinking. We asked professionals that on a 1 to 6 scale, what effect had the PUSE task on the mathematical and logical thinking (1- negative, 6- positive).

The average - based on the results - was 4.4, the deviation 0.9. We can say again that by the opinion of professionals, the PolyUni game had a positive effect on developing mathematical and logical skills. We have also considered the subjective opinions and we got the following wordings:

“They meet with variations and combinations during the game. Learning don’t have a “school feeling”, it is more like a freetime activity when they play with the set. It keeps their attention awake, it offers multiple opportunities, not just one solution. It makes the formulation of unique rules and games possible.”

“The Poliversum is built by three base forms and a scale shift layout of these, and combination of four colours. Thanks to this, it contains almost infinite variable opportunities. The game - thanks to the infinite number of combinations - develops logical and mathematical skills and teach the basics of the mathematical thinking in an indirect way. ”

“During the workshops, students’ logical skills were used; they can apply their mathematical knowledge.”

3.6.3 Social skills

Besides the cognitive and mathematical skills, we considered it important that the research explores social skills, too. As before, the participants marked the level of progression on a scale 1 to 6 (1-negative, 6-positive), then they justified their answers in open questions. Based on the results the average was 4.6, the deviation 0.9. According to sprofessionals, PolyUni develops social skills during the game because it contains tasks that require teamwork, project coordination and collaboration. This is supported by the statements of the professionals:

“It is highly suitable for developing collaboration, it teaches the acceptance of each other’s ideas.”

“They had to talk about which piece(s) they need to find for the statement to be true.”

“The collective problem solving and teamwork efficiency have grown.”

“Nowadays more and more children are struggling with learning difficulties; hyperactivity, lack of concentration, dyscalculia, and attention deficit disorder are common phenomena not just in elementary school. The Poly-Universe is a unique “mix”, which - with a special combination of art, mathematics and game - grabs attention, requires concentration, helps in experiencing freedom and in making learning easier and happier. The game is also a team building activity: if two or three kids works with a set of 24 pieces, sooner or later it is obvious that they need to unite the pieces and together can reach a common goal. Usually a leader emerges, and everyone gets involved.”

3.6.4 Attention

Last but not least, we asked professionals about the attention skill progression. Educators who lead the activities expressed their opinion on the development of their students' attention skill during the task with the PUSE on a scale 1 to 6. After that, they confirmed their opinion in open answers. Based on the results obtained the average was 4.2, the deviation is 0.8. The qualitative research section contains the following statements:

"As you must pay attention to shapes, colours and size at the same time, it is educational by all means."

"The tasks caught the attention for a while, but with longer or difficult tasks children's attention was distracted. The quantity of one set is too little as there are too many children for one set. One set for each pair would be better."

"Settling in this game demands patience and attention, which got better and better for the students."

"Primary school children - almost without exception - liked the game. Regardless of school or country, they knew immediately what they need to do with it. They felt the regularities in it. The progressively structured colour- and shape groups, the high degree of manuality, the contemplating thinking induce a constant challenge and explorative desire in the children, and also an uninterrupted and continuous sense of achievement. The direct physical activity, the emotional charge by the colours, the possibility of trying out many free variations without being evaluated, all help to keep their attention and stimulates endurance. Older students, mainly highschoolers have a biased attitude; they consider it as a childish game and there are always a a few of them who don't want to play with it. Even with guidance, overcoming the deadlock becomes a bigger and bigger challenge, and finally, they can solve the hardest tasks, which are usually more than an hour."

4. Summary and outlook

The goal of the research related to the PUSE project was to examine the effects of the PolyUni activities. During the research we used cognitive tests, online questionnaire and qualitative methods.

Among the studied fields there are some cognitive topics like visual perception, attention, mental rotation, and memory. Besides these, we also covered the topics of creativity, progression of social skills, and mathematical and logical thinking. Our research equally focused on students' performance and attitude and on the teachers' attitude who developed and guided the activities.

The received results partly confirms our hypothesis, and partly recommend further research on the outlined fields. Discernible positive effect showed up on the visual perception progress, which is firmly separable in the case of the experimental and control group, too. Regarding this, we can say that activities with the PolyUni devices largely helps visual perception, including the isolation and development of form-background skill. In the case of memory and mental rotation we didn't experienced differences. Although it doesn't mean that the device would not be suitable for developing these skills, but probably it requires a longer development program for tracking the differences. According to professionals, there is progression on the fields mentioned, which is also supported by the results mentioned above.

The research of the attitude of students didn't show significant differences between the pre-measurement and post-measurement, but it is worth keeping in mind that basically there were high avarages (3<) during the pre-measurement. Furthermore, it is important to mention that during the five month period students were influenced by other stimuli too, so their opinion was formed in multiple ways. Based on the opinions of professionals it can be we can say that during the activity children got richer with positive feelings and they view mathematics and tasks related to it in a different way.

All in all, we can say that further research is required to get a better insight in the nature of PUSE's effect. The five month development period demonstrated that the PUSE activity had a positive influence on visual perception, and strongly supported the emergence of positive attitude towards mathematics.

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